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# Non(anti)commutative $\mathcal{N} = (1, 1/2)$ supersymmetric $\mathrm{U}(1)$ gauge theory

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Abstract: We study a reduction of deformation parameters in non(anti)commutative $\mathcal{N} = 2$ harmonic superspace to those in non(anti)commutative $\mathcal{N} = 1$ superspace. By this reduction we obtain the exact gauge and supersymmetry transformations in the Wess-Zumino gauge of non(anti)commutative $\mathcal{N} = 2$ supersymmetric U(1) gauge theory defined in the deformed harmonic superspace. We also find that the action with the first order correction in the deformation parameter reduces to the one in the $\mathcal{N} = 1$ superspace by some field redefinition. We construct deformed $\mathcal{N} = (1, 1/2)$ supersymmetry in $\mathcal{N} = 2$ supersymmetric U(1) gauge theory in non(anti)commutative $\mathcal{N} = 1$ superspace.

Keywords: Superspaces, Extended Supersymmetry, Supersymmetric Effective Theories, Non-Commutative Geometry.



 

## Contents



## 1. Introduction

Supersymmetric field theories in deformed superspace [1, 2] have been recently attracted much attentions in view of studying superstring effective field theories on the D-branes with graviphoton background [3−5]. Non(anti)commutative superspace is a deformed superspace with nonanticommutative Grassmann coordinates and the $*$-product. Field theories in non(anti)commutative superspace is constructed in terms of superfields and have been extensively studied [6−8]–[16].

$\mathcal{N}=1$ super Yang-Mills theory in the non(anti)commutative $\mathcal{N}=1$ superspace has been formulated in [6], where the deformation preserves the chiral structure of the theory. As in the commutative case, one can take the Wess-Zumino(WZ) gauge for vector superfields to write down the deformed action in terms of component fields. Since supersymmetry transformation cannot keep the WZ gauge, one may perform additional gauge transformation to recover the WZ gauge. In the nonanticommutative case, this gauge transformation induces the terms which depends on the deformation parameter $C$. Therefore, even if we redefine the component fields such that these fields transform canonically under the gauge transformation, the supersymmetry transformations receive the deformation due to non(anti)commutativity.

In a previous paper [7], we wrote down the deformed action of $\mathcal{N}=1$ supersymmetric U($N$) gauge theory with matter fields in both fundamental and adjoint representations. We also examined invariance of the action under the $\mathcal{N}=1/2$ supersymmetry transformation. In particular, in the case of adjoint matter fields, we claimed that we could not find the (deformed) extended supersymmetry transformation and concluded that only the $\mathcal{N}=1/2$ supersymmetry is preserved. It was pointed out in [12], however, that this conclusion had discrepancy with the general argument concerning the symmetry of the deformed $\mathcal{N}=2$ superspace whose Poisson structure constructed by the chiral supercharges $Q^i_\alpha$ preserves at least the $\mathcal{N}=(1,0)$ supersymmetry of the theory generated by $Q^1_\alpha$, $Q^2_\alpha$. Moreover, it is natural to expect that the action defined in the deformed harmonic superspace leads to the one in the deformed $\mathcal{N}=1$ superspace by the reduction of deformation parameters of the harmonic superspace to the deformed $\mathcal{N}=1$ superspace. In this case, it was also argued in [12] that the deformation preserves the $\mathcal{N}=(1,1/2)$ supersymmetry generated by $Q^1$, $Q^2$ and $\bar{Q}_2$. The existence of $\mathcal{N}=(1,1/2)$ supersymmetry is partly supported by our recent work [13], in which the deformed $\mathcal{N}=(1,0)$ supersymmetry has been constructed in the $\mathcal{N}=2$ supersymmetric U(1) gauge theory in non(anti)commutative harmonic superspace with generic (non-singlet) deformations. Thus we expect $\mathcal{N}=(1,1/2)$ supersymmetry in the $\mathcal{N}=2$ supersymmetric U($N$) gauge theory in the non(anti)commutative $\mathcal{N}=1$ superspace.

In this paper, we will construct deformed $\mathcal{N}=(1,1/2)$ supersymmetry in $\mathcal{N}=2$ supersymmetric U(1) gauge theory in non(anti)commutative $\mathcal{N}=1$ superspace. We compare this theory to the one in non(anti) commutative $\mathcal{N}=2$ harmonic superspace obtained by the reduction to the deformed $\mathcal{N}=1$ superspace. By this reduction we find the exact gauge and supersymmetry transformations preserving the WZ gauge. We also find that these transformations and the $O(C)$ action in the WZ gauge defined in the deformed harmonic superspace reduces to the one in the $\mathcal{N}=1$ superspace by field redefinition.

This paper is organised as follows: In section 2, we review $\mathcal{N}=2$ supersymmetric U(1) gauge theory in non(anti)commutative $\mathcal{N}=1$ superspace and non(anti)commutative $\mathcal{N}=2$ harmonic superspace with generic non-singlet deformation parameters $C$. For the latter superspace we construct $\mathcal{N}=2$ supersymmetric U(1) gauge theory in this superspace at $O(C)$. We then study the reduction of the deformation parameters in non(anti)commutative harmonic superspace to the deformed $\mathcal{N}=1$ superspace. We find that the $O(C)$ action reduces to the one defined in the deformed $\mathcal{N}=1$ superspace and argue invariance under $\mathcal{N}=(1,0)$ supersymmetry. In section 3, we investigate the reduced theory in more detail. We construct explicitly the exact gauge and $\mathcal{N}=(1,1/2)$ supersymmetry transformations of the component fields in the WZ gauge, where detailed calculations are explained in two appendices. After the field redefinition, we find deformed $\mathcal{N}=(1,1/2)$ supersymmetry transformation which keeps the action in the deformed $\mathcal{N}=1$ superspace invariant. Section A is devoted to discussion and conclusion. In appendix A, we describe some useful reduction formulas of harmonic variables. The details of calculation of the exact $\mathcal{N} = (1, 1/2)$ supersymmetry transformation laws are presented in appendix B.

## 2. $\mathcal{N} = 2$ supersymmetric U(1) gauge theory in non(anti)commutative superspaces

In this section we review $\mathcal{N} = 2$ supersymmetric U(1) gauge theory in the deformed $\mathcal{N} = 1$ superspace [6, 7] and non(anti)commutative $\mathcal{N} = 2$ harmonic superspace [12–14].

### 2.1 The deformed $\mathcal{N} = 1$ superspace

Let $(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ be the supercoordinates of $\mathcal{N} = 1$ superspace[17]. Here $\mu = 0, 1, 2, 3$ are spacetime indices, $\alpha, \dot{\alpha} = 1, 2$ the spinor indices. We study spacetime with the Euclidean signature so that chiral and antichiral fermions transform independently. We may call this $\mathcal{N} = 1$ superspace as $\mathcal{N} = (1/2, 1/2)$ superspace as in [12]. The non(anti)commutative $\mathcal{N} = 1$ superspace is introduced by imposing nonanticommutativity for Grassmann coordinates $\theta^\alpha$:

$$\{\theta^\alpha, \theta^\beta\}_* = C^{\alpha\beta}\,, \tag{2.1}$$

whereas the chiral coordinates $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ commute with other coordinates and $\bar{\theta}^{\dot{\alpha}}$ (anti)commute. The $*$-product $f * g$ is defined by $f * g = f \exp(P) g$, where $P = -\frac{1}{2}\overleftarrow{Q}_\alpha C^{\alpha\beta}\overrightarrow{Q}_\beta$ defines the Poisson structure on the superspace. Since this $Q$-deformation preserves chirality, i.e. the Poisson structure $P$ commutes with the supercovariant derivatives $D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu$ and $\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^\mu)^{\dot{\alpha}}\partial_\mu$, we can define (anti)chiral superfields $\Phi$ ($\bar{\Phi}$) satisfying $\bar{D}_{\dot{\alpha}}\Phi = 0$ ($D_\alpha\bar{\Phi} = 0$). We can also introduce the vector superfield $V(y, \theta, \bar{\theta})$ for representing gauge fields. If we take the WZ gauge for the vector superfield, we are able to write down the Lagrangian in terms of component fields by replacing the product to the $*$-product. The component fields do not transform canonically under the gauge transformation due to the deformation parameter $C$. But in [6, 7], the field redefinition was found such that the component fields transform canonically under the gauge transformation.

For $\mathcal{N} = 2$ U(1) gauge theory, matter fields $\Phi$, $\bar{\Phi}$ in the adjoint representation (under the $*$-product) and the vector superfield $V$ in the WZ gauge are given by

$$\begin{aligned}
\Phi(y, \theta) &= A(y) + \sqrt{2}\theta\psi(y) + \bar{\theta}\bar{\theta}F(y),\\
\bar{\Phi}(\bar{y}, \bar{\theta}) &= \bar{A}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) + \bar{\theta}\bar{\theta}\left(\bar{F} + 2iC^{\mu\nu}\partial_\mu(\bar{A}v_\nu)\right)(\bar{y}),\\
V(y, \theta, \bar{\theta}) &= -\theta\sigma^\mu\bar{\theta}v_\mu(y) + i\theta\theta\bar{\theta}\bar{\lambda}(y) - i\bar{\theta}\bar{\theta}\theta^\alpha\left(\lambda_\alpha + \frac{1}{2}\varepsilon_{\alpha\beta}C^{\beta\gamma}(\sigma^\mu\bar{\lambda})_\gamma v_\mu\right)(y) +\\
&\quad + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D - i\partial^\mu v_\mu)(y)\,,
\end{aligned} \tag{2.2}$$

where $\bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$ is the anti-chiral coordinates. The deformed action is defined by

$$S = \int d^4x d^2\theta d^2\bar{\theta}\bar{\Phi} * e^V * \Phi * e^{-V} + \frac{1}{16g^2}\left(\int d^2\theta W^\alpha * W_\alpha + \int d^2\bar{\theta}\bar{W}_{\dot{\alpha}} * \bar{W}^{\dot{\alpha}}\right). \tag{2.3}$$

Here $g$ denotes the gauge coupling constant. The chiral and anti-chiral field strength are

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V\,, \qquad \bar{W}_{\dot\alpha} = \frac{1}{4}DDe^V\bar{D}_{\dot\alpha}e^{-V}\,, \tag{2.4}$$

where the multiplication of superfields is defined by using the $*$-product. After rescaling $V$ to $2gV$ and $C^{\alpha\beta}$ to $\frac{1}{2g}C^{\alpha\beta}$, this deformed action is expressed as

$$S = \int d^4x(\mathcal{L}^{(0)} + \mathcal{L}^{(1)})\,, \tag{2.5}$$

where

$$\mathcal{L}^{(0)} = -\frac{1}{4}v_{\mu\nu}(v^{\mu\nu} + \tilde{v}^{\mu\nu}) - i\bar\lambda\bar\sigma^\mu\partial_\mu\lambda + \frac{1}{2}D^2 - \partial^\mu\bar{A}\partial_\mu A - i\bar\psi\bar\sigma^\mu\partial_\mu\psi + \bar{F}F\,, \tag{2.6}$$

$$\mathcal{L}^{(1)} = -\frac{i}{2}C^{\mu\nu}v_{\mu\nu}(\bar\lambda\bar\lambda) + \sqrt{2}C^{\alpha\beta}\psi_\alpha(\sigma^\mu\bar\lambda)_\beta\partial_\mu\bar{A} + iC^{\mu\nu}v_{\mu\nu}\bar{A}F\,, \tag{2.7}$$

and $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$. This action is invariant under the gauge transformation $\delta_\lambda v_\mu = -\partial_\mu\lambda$. Undeformed theory has $\mathcal{N}=2$ extended supersymmetry, where only $\mathcal{N}=1$ symmetry generated by $Q_\alpha$ and $\bar{Q}_{\dot\alpha}$ is manifest in $\mathcal{N}=1$ superspace formalism. In deformed theory, the generator $\bar{Q}_{\dot\alpha}$ does not commute with the Poisson structure $P$ and is not a symmetry of the theory. On the other hand, $Q_\alpha$ generates a symmetry of the theory. Since $Q$ transformation does not preserve the WZ gauge, we need to do gauge transformation to retain the WZ gauge. The deformed supersymmetry transformation is

$$\begin{aligned}
\delta^*_\xi A &= \sqrt{2}\xi\psi\,, \qquad \delta^*_\xi\bar{A} = 0\,,\\
\delta^*_\xi\psi_\alpha &= \sqrt{2}\xi_\alpha F\,, \qquad \delta^*_\xi\bar\psi_{\dot\alpha} = -i\sqrt{2}(\xi\sigma^\mu)_{\dot\alpha}\partial_\mu\bar{A}\,,\\
\delta^*_\xi F &= 0\,, \qquad \delta^*_\xi\bar{F} = i\sqrt{2}(\xi\sigma^\mu\partial_\mu\bar\psi) - 2C^{\alpha\beta}\xi_\alpha\sigma^\mu_{\beta\dot\alpha}\partial_\mu(\bar\lambda^{\dot\alpha}\bar{A})\,,\\
\delta^*_\xi v_\mu &= i\xi\sigma_\mu\bar\lambda\,,\\
\delta^*_\xi\lambda_\alpha &= (\sigma^{\mu\nu}\xi)_\alpha\left\{v_{\mu\nu} + \frac{i}{2}C_{\mu\nu}(\bar\lambda\bar\lambda)\right\} + i\xi_\alpha D\,, \qquad \delta^*_\xi\bar\lambda_{\dot\alpha} = 0\,,\\
\delta^*_\xi D &= -(\xi\sigma^\mu\partial_\mu\bar\lambda)\,.
\end{aligned} \tag{2.8}$$

Remaining $\mathcal{N}=1$ supersymmetry, however, is not manifest in this formalism. In order to obtain this supersymmetry transformation more systematically, it is convenient to introduce non(anti)commutative extended superspace. In the next section, we will discuss $\mathcal{N}=2$ supersymmetric U(1) gauge theory in non(anti)commutative harmonic superspace.

### 2.2 Non(anti)commutative $\mathcal{N}=2$ harmonic superspace

Next we review non(anti)commutative deformation of $\mathcal{N}=2$ harmonic superspace and $\mathcal{N}=2$ supersymmetric U(1) gauge theory in this superspace [13, 14]. Let $(x^\mu, \theta^i_\alpha, \bar\theta^i_{\dot\alpha})$ be the coordinates of $\mathcal{N}=2$ (rigid) superspace. The index $i=1,2$ labels the doublet of the $\mathrm{SU}(2)_R$ $R$-symmetry. The supersymmetry generators $Q^i_\alpha$, $\bar{Q}_{\dot\alpha i}$ and the supercovariant derivatives $D^i_\alpha$, $\bar{D}_{\dot\alpha i}$ are defined by

$$\begin{aligned}
Q^i_\alpha &= \frac{\partial}{\partial\theta^\alpha_i} - i(\sigma^\mu)_{\alpha\dot\alpha}\bar\theta^{\dot\alpha i}\frac{\partial}{\partial x^\mu}\,, \qquad \bar{Q}_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} + i\theta^\alpha_i(\sigma^\mu)_{\alpha\dot\alpha}\frac{\partial}{\partial x^\mu}\,,\\
D^i_\alpha &= \frac{\partial}{\partial\theta^\alpha_i} + i(\sigma^\mu)_{\alpha\dot\alpha}\bar\theta^{\dot\alpha i}\frac{\partial}{\partial x^\mu}\,, \qquad \bar{D}_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} - i\theta^\alpha_i(\sigma^\mu)_{\alpha\dot\alpha}\frac{\partial}{\partial x^\mu}\,.
\end{aligned} \tag{2.9}$$

The $\mathcal{N}=2$ harmonic superspace [18] is introduced by adding the harmonic variables $u_i^\pm$ to the $\mathcal{N}=2$ superspace coordinates. The variables $u_i^\pm$ form an SU(2) matrix and satisfy the conditions $u^{+i}u_i^-=1$ and $\overline{u^{+i}}=u_i^-$. The completeness condition for $u_i^\pm$ reads $u_i^+u_j^- - u_j^+u_i^- = \epsilon_{ij}$. Using $u_i^\pm$, the $\mathrm{SU}(2)_R$ indices can be projected into two parts with $\pm 1$ $\mathrm{U}(1)(\subset \mathrm{SU}(2)_R)$ charges. For example, we define the supercovariant derivatives $D_\alpha^\pm$ and $\bar{D}_{\dot\alpha}^\pm$ by $D_\alpha^\pm = u_i^\pm D_\alpha^i$, $\bar{D}_{\dot\alpha}^\pm = u_i^\pm \bar{D}_{\dot\alpha}^i$. $D_\alpha^i$ is solved by $D_\alpha^\pm$ such as $D_\alpha^\pm = u_i^+ D_\alpha^- - u_i^- D_\alpha^+$ with the help of the completeness condition. In the harmonic superspace formalism, an important ingredient is an analytic superfield rather than the $\mathcal{N}=2$ chiral superfield. An analytic superfield $\Phi(x,\theta,\bar\theta,u)$ is defined by $D_\alpha^+\Phi = \bar{D}_{\dot\alpha}^+\Phi = 0$. It is convenient to write this analytic superfield in terms of analytic basis: $x_A^\mu = x^\mu - i(\theta^i\sigma^\mu\bar\theta^j + \theta^j\sigma^\mu\bar\theta^i)u_i^+u_j^-$, $\theta_\alpha^\pm = u_i^\pm\theta_\alpha^i$ and $\bar\theta_{\dot\alpha}^\pm = u_i^\pm\bar\theta_{\dot\alpha}^i$. In this basis, an analytic superfield $\Phi$ is functions of $(x_A^\mu,\theta^+,\bar\theta^+,u)$: $\Phi = \Phi(x_A^\mu,\theta^+,\bar\theta^+,u)$.

We now introduce the nonanticommutativity in the $\mathcal{N}=2$ harmonic superspace by using the $*$-product:

$$f * g(\theta) = f(\theta)\exp(P)g(\theta), \quad P = -\frac{1}{2}\overleftarrow{Q_\alpha^i}C_{ij}^{\alpha\beta}\overrightarrow{Q_\beta^j}, \tag{2.10}$$

where $C_{ij}^{\alpha\beta}$ is some constants. With this $*$-product, we have following (anti)commutation relations:

$$\{\theta_i^\alpha,\theta_j^\beta\}_* = C_{ij}^{\alpha\beta}, \ [x_L^\mu,x_L^\nu]_* = [x_L^\mu,\theta_i^\alpha]_* = [x_L^\mu,\bar\theta^{\dot\alpha i}]_* = 0, \ \{\bar\theta^{\dot\alpha i},\bar\theta^{\dot\beta j}\}_* = \{\bar\theta^{\dot\alpha i},\theta_j^\alpha\}_* = 0, \tag{2.11}$$

where $x_L^\mu \equiv x^\mu + i\theta_i\sigma^\mu\bar\theta^i$ are $\mathcal{N}=2$ chiral coordinates. The deformation parameter $C_{ij}^{\alpha\beta}$ is symmetric under the exchange of pairs of indices $(\alpha i)$,$(\beta j)$: $C_{ij}^{\alpha\beta} = C_{ji}^{\beta\alpha}$. We decompose the nonanticommutative parameter $C_{ij}^{\alpha\beta}$ into the symmetric and antisymmetric parts with respect to the SU(2) indices, such as

$$C_{ij}^{\alpha\beta} = C_{(ij)}^{\alpha\beta} + \frac{1}{4}\epsilon_{ij}\varepsilon^{\alpha\beta}C_s. \tag{2.12}$$

Here we denote $A_{(i_1\dots i_n)}$ by the symmetrized sum of $A_{i_1\dots i_n}$ over indices $i_1,\dots,i_n$. $C_{ij}^{\alpha\beta}$ with zero $C_{(ij)}^{\alpha\beta}$ corresponds to the singlet deformation [12, 15, 16]. Since $P$ commutes with the supercovariant derivatives $D$, the chiral structure is preserved by this deformation. Since we will consider the non-singlet deformation in the following, we put $C_s = 0$.

We now construct the action of $\mathcal{N}=2$ supersymmetric U(1) gauge theory in this non(anti)commutative superspace. We introduce an analytic superfield $V^{++}(\zeta,u)$ with $\zeta = (x_A^\mu,\theta^+,\bar\theta^+)$ by covariantizing the harmonic derivative $D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} - 2i\theta^+\sigma^\mu\bar\theta^+\frac{\partial}{\partial x_A^\mu} + \theta^{+\alpha}\frac{\partial}{\partial\theta^{-\alpha}} + \bar\theta^{+\dot\alpha}\frac{\partial}{\partial\bar\theta^{-\dot\alpha}} \to \nabla^{++} = D^{++} + iV^{++}$. Generalizing the construction in [19, 20], the action is given by

$$S_* = \frac{1}{2}\sum_{n=2}^\infty \int d^4x d^8\theta du_1\dots du_n \frac{(-i)^n}{n}\frac{V^{++}(1)*\cdots*V^{++}(n)}{(u_1^+u_2^+)\cdots(u_n^+u_1^+)}, \tag{2.13}$$

where $V^{++}(i) = V^{++}(\zeta_i,u_i)$, $\zeta_i = (x_A,\theta_i^+,\bar\theta_i^+)$ and $d^8\theta = d^4\theta^+d^4\theta^-$ with $d^4\theta^\pm = d^2\theta^\pm d^2\bar\theta^\pm$. The harmonic integral is defined by the rules: (i) $\int du f(u) = 0$ for $f(u)$ with non-zero U(1) charge. (ii) $\int du 1 = 1$. (iii) $\int du u_{(i_1}^+\cdots u_{i_n}^+u_{j_1}^-\cdots u_{j_n)}^- = 0$, $(n \geq 1)$. The action (2.13) is invariant under the gauge transformation

$$\delta^*_\Lambda V^{++} = -D^{++}\Lambda + i[\Lambda, V^{++}]_* \,, \tag{2.14}$$

with an analytic superfield $\Lambda$. The generic superfield $V^{++}(\zeta, u)$ includes infinitely many auxiliary fields. Most of these fields are gauged away except the lowest component fields in the harmonic expansion. One can take the WZ gauge

$$\begin{aligned} V^{++}_{WZ}(x_A, \theta^+, \bar\theta^+, u) \,=\, & -i\sqrt{2}(\theta^+)^2\bar\phi(x_A) + i\sqrt{2}(\bar\theta^+)^2\phi(x_A) - 2i\theta^+\sigma^\mu\bar\theta^+ A_\mu(x_A) + \\ & + 4(\bar\theta^+)^2\theta^+\psi^i(x_A)u^-_i - 4(\theta^+)^2\bar\theta^+\bar\psi^i(x_A)u^-_i + \\ & + 3(\theta^+)^2(\bar\theta^+)^2 D^{ij}(x_A)u^-_i u^-_j \,, \end{aligned} \tag{2.15}$$

which is convenient to study the theory in the component formalism.

The component action $S_*$ in the WZ gauge can be expanded in a power series of the deformation parameter $C$. In [14], we have computed the $O(C)$ action explicitly. The quadratic part $S_{*,2}$ in $S_*$ is the same as the commutative one:

$$S_{*,2} = \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{\mu\nu}\tilde F^{\mu\nu} + \phi\partial^2\bar\phi - i\psi^i\sigma^\mu\partial_\mu\bar\psi_i + \frac{1}{4}D^{ij}D_{ij} \right\}. \tag{2.16}$$

The cubic part $S_{*,3}$ in $S_*$ is of order $O(C)$ and given by

$$\begin{aligned} S_{*,3} \,=\, \int d^4x \Big[ & -\frac{2\sqrt{2}}{3} i C^{\alpha\beta}_{(ij)}\psi^i_\alpha(\sigma^\nu\partial_\nu\bar\psi^j)_\beta\bar\phi - 2\sqrt{2}iC^{\alpha\beta}_{(ij)}\psi^i_\alpha(\sigma^\nu\bar\psi^j)_\beta\partial_\nu\bar\phi + \\ & + \frac{2}{3}iC^{\alpha\beta}_{(ij)}A_\mu(\sigma^\mu\bar\psi^i)_\alpha(\sigma^\nu\partial_\nu\bar\psi^j)_\beta - iC^{\mu\nu}_{(ij)}\bar\psi^i\bar\psi^j F_{\mu\nu} + \\ & + \sqrt{2}C^{\mu\nu}_{(ij)}D^{ij}A_\mu\partial_\nu\bar\phi + \frac{1}{\sqrt{2}}C^{\mu\nu}_{(ij)}D^{ij}F_{\mu\nu}\bar\phi \Big] . \end{aligned} \tag{2.17}$$

Note that here we have already dropped the $C_s$ dependent terms. We will refer $S_{*,2} + S_{*,3}$ as the $O(C)$ action.

In the commutative case, the gauge parameter $\Lambda = \chi(x_A)$ preserves the WZ gauge and gives rise to the gauge transformation for component fields. In the non(anti)commutative case, however, the gauge transformation (2.14) with the same gauge parameter does not preserve the WZ gauge because of the $C$-dependent terms arising from the commutator. In order to preserve the WZ gauge, one must include the $C$-dependent terms. The gauge parameter is shown to take the form

$$\begin{aligned} \lambda_C(\zeta, u) \,=\, & \chi(x_A) + \theta^+\sigma^\mu\bar\theta^+\lambda^{(-2)}_\mu(x_A, u; C) + (\bar\theta^+)^2\lambda^{(-2)}(x_A, u; C) + \\ & + (\bar\theta^+)^2\theta^{+\alpha}\lambda^{(-3)}_\alpha(x_A, u; C) + (\theta^+)^2(\bar\theta^+)^2\lambda^{(-4)}(x_A, u; C) \,, \end{aligned} \tag{2.18}$$

which has been determined by solving the WZ gauge preserving conditions expanded in harmonic modes [14]. The gauge transformation is also fully determined, which reads

$$\begin{aligned} \delta^*_{\lambda_C} A_\mu \,&=\, -\partial_\mu\chi + O(C^2) \,, \\ \delta^*_{\lambda_C}\phi \,&=\, O(C^2) \,, \\ \delta^*_{\lambda_C}\psi_{\alpha i} \,&=\, \frac{2}{3}(\varepsilon C_{(ij)}\sigma^\mu\bar\psi^j)_\alpha\,\partial_\mu\chi + O(C^2) \,, \\ \delta^*_{\lambda_C} D_{ij} \,&=\, 2\sqrt{2}C^{\mu\nu}_{(ij)}\partial_\mu\chi\partial_\nu\bar\phi + O(C^2) \,, \\ \delta^*_{\lambda_C}(\text{others}) \,&=\, 0 \,. \end{aligned} \tag{2.19}$$

The $O(C)$ action is invariant under the $O(C)$ gauge transformation (2.19).



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These gauge transformations are not canonical. But if we redefine the component fields such as

$$\begin{aligned}
\hat{A}_\mu &= A_\mu + O(C^2), \\
\hat{\phi} &= \phi + O(C^2)\,, \qquad \hat{\bar{\phi}} = \bar{\phi}\,, \\
\hat{\psi}_{\alpha i} &= \psi_{\alpha i} + \frac{2}{3}(\varepsilon C_{(ij)}\sigma^\mu \bar{\psi}^j)_\alpha A_\mu + O(C^2)\,, \qquad \hat{\bar{\psi}}^{\dot{\alpha}} = \bar{\psi}^{\dot{\alpha}}\,, \\
\hat{D}_{ij} &= D_{ij} + 2\sqrt{2}C^{\mu\nu}_{(ij)} A_\mu \partial_\nu \bar{\phi} + O(C^2)\,,
\end{aligned} \tag{2.20}$$

the newly defined fields are shown to transform canonically: $\delta^*_{\lambda_C}\hat{A}_\mu = -\partial_\mu \chi$, $\delta^*_{\lambda_C}(\text{others}) = 0$. In terms of redefined fields, the $O(C)$ action can be written as

$$\begin{aligned}
S_{*,2} + S_{*,3} = \int d^4x \Big[ &-\frac{1}{4}\hat{F}_{\mu\nu}(\hat{F}^{\mu\nu} + \tilde{\hat{F}}^{\mu\nu}) + \hat{\phi}\partial^2\hat{\bar{\phi}} - i\hat{\psi}^i \sigma^\mu \partial_\mu \hat{\bar{\psi}}_i + \frac{1}{4}\hat{D}^{ij}\hat{D}_{ij} - \\
&- 2\sqrt{2}iC^{\alpha\beta}_{(ij)}\hat{\psi}^i_\alpha(\sigma^\mu \hat{\bar{\psi}}^j)_\beta \partial_\mu \hat{\bar{\phi}} - \frac{2\sqrt{2}}{3} i C^{\alpha\beta}_{(ij)}\hat{\psi}^i_\alpha(\sigma^\mu\partial_\mu\hat{\bar{\psi}}^j)_\beta \hat{\bar{\phi}} - \\
&- iC^{\mu\nu}_{(ij)}\hat{\bar{\psi}}^i \hat{\bar{\psi}}^j \hat{F}_{\mu\nu} + \frac{1}{\sqrt{2}} C^{\mu\nu}_{(ij)}\hat{D}^{ij}\hat{F}_{\mu\nu}\hat{\bar{\phi}} + O(C^2)\Big]\,,
\end{aligned} \tag{2.21}$$

where $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$.

The $O(C)$ deformed $\mathcal{N} = 2$ supersymmetry transformation of the component fields in the WZ gauge is determined in a similar way[13]. After the field redefinition (2.20), the transformation is given by

$$\begin{aligned}
\delta^*_\xi \hat{\phi} &= -\sqrt{2}i\xi^i \hat{\psi}_i - \frac{8}{3}i(\xi^j \varepsilon C_{(jk)}\hat{\psi}^k)\hat{\bar{\phi}} + O(C^2)\,, \\
\delta^*_\xi \hat{\bar{\phi}} &= 0\,, \\
\delta^*_\xi \hat{A}_\mu &= i\xi^i \sigma_\mu \hat{\bar{\psi}}_i + 2\sqrt{2}i(\xi^j \varepsilon C_{(jk)}\sigma_\mu \hat{\bar{\psi}}^k)\hat{\bar{\phi}} + O(C^2)\,, \\
\delta^*_\xi \hat{\psi}^{\alpha i} &= -(\xi^i \sigma^{\mu\nu})^\alpha \hat{F}_{\mu\nu} - \hat{D}^{ij}\xi^\alpha_j - i(\xi^i\sigma_{\mu\nu})^\alpha C^{\mu\nu}_{(jk)}(\hat{\bar{\psi}}^j \hat{\bar{\psi}}^k) + 2\sqrt{2}\hat{D}^{(ij}(\xi^{k)}\varepsilon C_{(jk)})^\alpha \hat{\bar{\phi}} - \\
&\quad - \left\{ 2\sqrt{2}(\xi^j \varepsilon C_{(jk)}\sigma^{\mu\nu})^\alpha + \frac{2\sqrt{2}}{3}(\xi^j \sigma^{\mu\nu}\varepsilon C_{(jk)})^\alpha + \sqrt{2}C^{\mu\nu}_{(jk)}\xi^{\alpha j} \right\} \epsilon^{ki}\hat{\bar{\phi}}\hat{F}_{\mu\nu} + O(C^2)\,, \\
\delta^*_\xi \hat{\bar{\psi}}^i_{\dot{\alpha}} &= +\sqrt{2}(\xi^i \sigma^\nu)_{\dot{\alpha}}\partial_\nu \hat{\bar{\phi}} + 2(\xi^j \varepsilon C_{(jk)}\sigma^\nu)_{\dot{\alpha}}\partial_\nu(\hat{\bar{\phi}}^2)\epsilon^{ki} + O(C^2)\,, \\
\delta^*_\xi \hat{D}^{ij} &= -2i\xi^{(i}\sigma^\nu \partial_\nu \hat{\bar{\psi}}^{j)} - 6\sqrt{2}i\epsilon^{k(l}\partial_\nu \times \\
&\quad \times \left\{ (\xi^i \varepsilon C_{(kl)}\sigma^\nu \hat{\bar{\psi}}^{j)})\hat{\bar{\phi}} \right\} + 2\sqrt{2}i\epsilon^{il}\epsilon^{jm}(\xi^k \varepsilon C_{(lm)}\sigma^\nu \hat{\bar{\psi}}_k)\partial_\nu \hat{\bar{\phi}} + O(C^2).
\end{aligned} \tag{2.22}$$

### 2.3 Reduction to the deformed $\mathcal{N} = 1$ superspace

Since the deformed $\mathcal{N} = 2$ action (2.5) in the deformed $\mathcal{N} = 1$ superspace contains only $O(C)$ corrections, it is important to compare $\mathcal{N} = 1$ superspace formalism to non(anti)-commutative harmonic superspace formalism. We will examine this correspondence in terms of component fields because it is difficult to find the relationship between $(\Phi,\bar{\Phi},V)$ and $V^{++}_{WZ}$ explicitly at the superfield level.

We impose the condition for the deformation parameters $C^{\alpha\beta}_{ij}$ such as

$$C^{\alpha\beta}_{ij} = C^{\alpha\beta}_{11}\delta^1_i \delta^1_j\,. \tag{2.23}$$

This reduction of the parameters implies that only the $\theta^i_\alpha$ coordinates are nonanticommutative and $\theta^2_\alpha$ are ordinary Grassmann coordinates. Then the action (2.21) with redefined component fields becomes

$$
\begin{aligned}
S_{*,2}+S_{*,3} = \int d^4x \Big[ & -\frac{1}{4}\hat{F}_{\mu\nu}(\hat{F}^{\mu\nu}+\tilde{\hat{F}}^{\mu\nu}) - i\hat{\psi}^i\sigma^\mu\partial_\mu\bar{\hat{\psi}}_i + \hat{\phi}\partial^\mu\partial_\mu\bar{\hat{\phi}} + \frac{1}{4}\hat{D}_{ij}\hat{D}^{ij} - \\
& - \frac{2\sqrt{2}}{3} iC^{\alpha\beta}_{11}\hat{\psi}^1_\alpha(\sigma^\nu\partial_\nu\bar{\hat{\psi}}^1)_\beta\bar{\hat{\phi}} - 2\sqrt{2}iC^{\alpha\beta}_{11}\hat{\psi}^1_\alpha(\sigma^\nu\bar{\hat{\psi}}^1)_\beta\partial_\nu\bar{\hat{\phi}} - \\
& - iC^{\mu\nu}_{11}\bar{\hat{\psi}}^1\bar{\hat{\psi}}^1\hat{F}_{\mu\nu} + \frac{1}{\sqrt{2}}C^{\mu\nu}_{11}\hat{D}^{11}\hat{F}_{\mu\nu}\bar{\hat{\phi}}\Big]. \qquad (2.24)
\end{aligned}
$$

Since both component fields $(\hat{\phi}, \bar{\hat{\phi}}, \hat{\psi}^i, \bar{\hat{\psi}}^i, \hat{D}^{ij})$ and $(A, \bar{A}, \psi, \bar{\psi}, \lambda, \bar{\lambda}, v_\mu, F, \bar{F}, D)$ transform canonically under the gauge transformation, these fields must be related to each other in gauge invariant way. By comparing (2.5) and (2.24), we find

$$
\begin{aligned}
& v_\mu = \hat{A}_\mu, \quad A = -i\hat{\phi}, \qquad \bar{A} = i\bar{\hat{\phi}}, \\
& D = i\hat{D}^{12}, \qquad F = -\frac{1}{\sqrt{2}}\hat{D}^{11}, \qquad \bar{F} = -\frac{1}{\sqrt{2}}\left(\hat{D}^{22} - \sqrt{2}C^{\mu\nu}_{11}\hat{F}_{\mu\nu}\bar{\hat{\phi}}\right), \\
& \lambda^\beta = \hat{\psi}^{2\beta} - \frac{2\sqrt{2}}{3}C^{\alpha\beta}_{11}\hat{\psi}^1_\alpha\bar{\hat{\phi}}, \qquad \psi = \hat{\psi}^1, \qquad \bar{\psi} = \bar{\hat{\psi}}_1, \quad \bar{\lambda} = \bar{\hat{\psi}}_2. \qquad (2.25)
\end{aligned}
$$

where the deformation parameter $C^{\alpha\beta}_{11}$ is related to $C^{\alpha\beta}$ by

$$
C^{\alpha\beta}_{11} = \frac{1}{2}C^{\alpha\beta}. \qquad (2.26)
$$

Note that for $C=0$ we get the field redefinition for undeformed theory.

We now consider the deformed supersymmetry transformations. Taking the Grassmann parameters as $(\xi^1, \xi^2) = (-\eta, \xi)$ and using the supersymmetry transformations (2.22) and identifications (2.25), it is shown that the transformation associated with $\xi$ becomes (2.8). The $Q^2$ supersymmetry transformation associated with the parameter $\eta$ would give a remaining transformation. But it turns out that $O(C)$ transformation with the same identification does not keep the action invariant. It is necessary to introduce $O(C^2)$ correction to the $Q^2$ supersymmetry transformation. We found that the following transformation keep the action invariant:

$$
\begin{aligned}
\delta^*_\eta A &= \sqrt{2}\eta\lambda - 2i\eta\varepsilon C\psi\bar{A}, \qquad \delta^*_\eta\bar{A} = 0, \\
\delta^*_\eta\psi^\alpha &= (\eta\sigma_{\mu\nu})^\alpha\left\{v^{\mu\nu} + \frac{i}{2}C^{\mu\nu}(\bar{\lambda}\bar{\lambda} - 2F\bar{A})\right\} + i\eta^\alpha D, \qquad \delta^*_\eta\bar{\psi}_{\dot\alpha} = -(\eta\varepsilon C\sigma^\nu)_{\dot\alpha}\partial_\nu(\bar{A}^2), \\
\delta^*_\eta F &= \sqrt{2}i\eta\sigma^\nu\partial_\nu\bar{\lambda}, \qquad \delta^*_\eta\bar{F} = 2\sqrt{2}i\det C\eta\sigma^\nu\partial_\nu(\bar{\lambda}\bar{A}^2), \\
\delta^*_\eta v_\mu &= -i\eta\sigma_\mu\bar{\psi} + \sqrt{2}(\eta\varepsilon C\sigma_\mu\bar{\lambda})\bar{A}, \\
\delta^*_\eta\lambda^\alpha &= \sqrt{2}\eta^\alpha\bar{F} + \sqrt{2}(\eta\varepsilon C)^\alpha\bar{A}D - \sqrt{2}i(\eta\sigma^{\mu\nu}\varepsilon C)^\alpha v_{\mu\nu}\bar{A} + \sqrt{2}\det C\eta^\alpha\bar{\lambda}\bar{\lambda}\bar{A}, \\
\delta^*_\eta\bar{\lambda}_{\dot\alpha} &= -\sqrt{2}i(\eta\sigma^\nu)_{\dot\alpha}\partial_\nu\bar{A}, \\
\delta^*_\eta D &= -\eta\sigma^\nu\partial_\nu\bar{\psi} + \sqrt{2}i\eta\varepsilon C\sigma^\nu\partial_\nu(\bar{\lambda}\bar{A}), \qquad (2.27)
\end{aligned}
$$

where $(\varepsilon C)_\alpha{}^\beta \equiv \varepsilon_{\alpha\gamma}C^{\gamma\beta}$. Hence we have found the $\mathcal{N}=(1,0)$ supersymmetry of the $\mathcal{N}=2$ action in the deformed $\mathcal{N}=1$ superspace.

In (2.27), we add $O(C^2)$ correction to the transformation by hand. This correction is not unique because the action (2.5) is invariant under the transformation $\tilde{\delta}_\eta$ with

$$\begin{aligned}\tilde{\delta}_\eta \lambda_\alpha &= \eta_\alpha f_1(\bar{A}) F\,,\\ \tilde{\delta}_\eta \bar{F} &= i f_1(\bar{A})(\eta\sigma^\mu \partial_\mu \bar{\lambda})\,,\end{aligned} \tag{2.28}$$

for arbitrary function $f_1(\bar{A})$ of $\bar{A}$. In the next section, we will show that we are able to construct exact supersymmetry transformation for the reduced deformation parameter (2.23). If we use the field redefinition (2.25), we can fix $f_1(\bar{A}) = 0$. The result is shown to be exactly equal to (2.27).

We note also that $\bar{Q}_2$ supersymmetry transformation commutes with the Poisson structure $P$. The theory is expected to have $\mathcal{N} = (1, 1/2)$ extended supersymmetry [12]. In the next section we will construct the exact $\mathcal{N} = (1, 1/2)$ supersymmetry transformation in the framework of the harmonic superspace formalism.

## 3. $\mathcal{N} = (1,\ 1/2)$ supersymmetry

In this section, we will present the $\mathcal{N} = (1,\ 1/2)$ supersymmetry transformation generated by $Q^1$, $Q^2$ and $\bar{Q}_2$, within the harmonic superspace formalism. Under the restriction of the deformation parameter (2.23), we can determine the $\mathcal{N} = (1,\ 1/2)$ supersymmetry transformation laws exactly. The main concern is the contribution from the deformed gauge transformation to retain the WZ gauge. Although the calculation is much simpler than the case of the generic deformation parameter, it turns out to be considerably lengthy even under the restriction. On the other hand, one finds that the determination of the exact gauge transformation laws is accomplished in a similar way but is much easier than the supersymmetry. Therefore, in order to illustrate how to determine the exact transformations, first we derive in section 3.1 the exact gauge transformation laws in [14], restricting ourselves to the reduced deformation parameter (2.23). The exact $\mathcal{N} = (1,\ 1/2)$ supersymmetry transformation is given in section 3.2, though the details of the actual calculation are presented in appendix B. From the exact gauge transformation, we can find a field redefinition that leads to the canonical component gauge transformation. We also give the $\mathcal{N} = (1,\ 1/2)$ transformation after this field redefinition.

### 3.1 Determination of the exact transformation laws

In this subsection, in order to demonstrate how to determine the exact transformation laws under the restriction (2.23), we will derive the exact gauge transformation laws in [14]. We will denote the analytic gauge parameter as

$$\begin{aligned}\Lambda(\zeta, u) = {}& \chi(x_A) + \bar{\theta}^+_{\dot{\alpha}} \lambda^{(0,1)\dot{\alpha}}(x_A, u) + \theta^{+\alpha} \lambda^{(1,0)}_\alpha(x_A, u) + (\bar{\theta}^+)^2 \lambda^{(0,2)}(x_A, u) +\\ & + (\theta^+)^2 \lambda^{(2,0)}(x_A, u) + \theta^+ \sigma^\mu \bar{\theta}^+ \lambda^{(1,1)}_\mu(x_A, u) + (\bar{\theta}^+)^2 \theta^{+\alpha} \lambda^{(1,2)}_\alpha(x_A, u) +\\ & + (\theta^+)^2 \bar{\theta}^+_{\dot{\alpha}} \lambda^{(2,1)\dot{\alpha}}(x_A, u) + (\theta^+)^2 (\bar{\theta}^+)^2 \lambda^{(2,2)}(x_A, u)\,,\end{aligned} \tag{3.1}$$

where $\lambda^{(n,m)}(x_A, u)$ is the $(\theta^+)^n (\bar{\theta}^+)^m$-component.

The equations to determine the deformed gauge transformation are given in [14]:

$$
\begin{aligned}
-2i\delta^*_\Lambda A_\mu &= 2i\partial_\mu\chi + 2\sqrt{2}iC^{(+-)\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\partial_\nu\chi\ \bar\phi - \\
&\quad - \partial^{++}\lambda^{(1,1)}_\mu - \sqrt{2}C^{++\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_\nu\bar\phi\,, && (3.2)\\
\sqrt{2}i\delta^*_\Lambda\phi &= -2iC^{(+-)\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\partial_\nu\chi\ A^\mu - \\
&\quad - C^{++\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_\nu A^\mu - \partial^{++}\lambda^{(0,2)}\,, && (3.3)\\
3\delta^*_\Lambda D^{ij}u^-_i u^-_j &= 4\sqrt{2}C^{--\mu\nu}\partial_\mu\chi\ \partial_\nu\bar\phi - \\
&\quad - i\partial^\mu\lambda^{(1,1)}_\mu - \sqrt{2}iC^{(+-)\alpha\beta}(\sigma^\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\partial_\nu(\lambda^{(1,1)}_\mu\bar\phi) - \partial^{++}\lambda^{(2,2)}\,, && (3.4)\\
4\delta^*_\Lambda\psi^i_\alpha u^-_i &= -4C^{(+-)\beta\gamma}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\beta\gamma}\partial_\nu\chi\ (\sigma^\mu\bar\psi^i)_\alpha u^-_i - 2iC^{++\beta\gamma}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\beta\gamma}\lambda^{(1,1)}_\nu(\sigma^\mu\bar\psi^i)_\alpha u^-_i - \\
&\quad - \partial^{++}\lambda^{(1,2)}_\alpha - 2\sqrt{2}(\varepsilon C^{++}\lambda^{(1,2)})_\alpha\bar\phi\,. && (3.5)
\end{aligned}
$$

Here we have already set $C_s = 0$. The transformation laws for $\bar\phi$ and $\bar\psi_i$ are not deformed, because their equations are not affected by the $*$-product.

Now we take into account the restriction (2.23). We have $C^{(+-)\alpha\beta} = C^{\alpha\beta}_{11}u^{+1}u^{-1}$ and so on. First we find that eq.(3.2) is solved in terms of power series of $u^{+1}$ and $u^{-1}$ by

$$
\begin{aligned}
\delta^*_\Lambda A_\mu &= -\partial_\mu\chi\,, && (3.6)\\
\lambda^{(1,1)}_\mu &= \sum_{n=1}^{\infty}\lambda^{(n)}_\mu\ (u^{+1})^{n-1}(u^{-1})^{n+1}\,, && (3.7)
\end{aligned}
$$

where

$$
\lambda^{(n)}_\mu \equiv -i\frac{(-2\sqrt{2})^n}{(n+1)!}(\overbrace{C_{11}\varepsilon C_{11}\cdots\varepsilon C_{11}}^{n})^{\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\bar\phi^n\partial_\nu\chi\,. \tag{3.8}
$$

Note that the harmonic expansion of $(u^{+1})^n(u^{-1})^m$ is

$$
(u^{+1})^n(u^{-1})^m = \overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-1}\cdots u^{-1)}}^{m} = \delta^1_{(i_1}\cdots\delta^1_{i_n}\delta^1_{j_1}\cdots\delta^1_{j_m)}u^{+(i_1}\cdots u^{+i_n}u^{-j_1}\cdots u^{-j_m)}\,. \tag{3.9}
$$

Eq. (3.6) is directly checked by substituting $\lambda^{(1,1)}_\mu$ into the right hand side of eq. (3.2), noting that $\lambda^{(n)}_\mu$ obeys

$$
C^{\alpha\beta}_{11}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\lambda^{(n)}_\nu\bar\phi = -\frac{n+2}{\sqrt{2}}\lambda^{(n+1)}_\mu \qquad (n\geq 1)\,. \tag{3.10}
$$

Substituting eq. (3.7) into (3.3) , we find that all the terms in the right hand side of eq. (3.3) is gauged away with $\lambda^{(2,0)}$, because every term has at least one $u^+$. This leads to

$$
\delta^*_\Lambda\phi = 0\,. \tag{3.11}
$$

Although the precise form of $\lambda^{(0,2)}$ is not needed, it is determined as

$$
\begin{aligned}
\lambda^{(0,2)} &= -iC^{\alpha\beta}_{11}(\sigma^\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\partial_\nu\chi A_\mu\ (u^{-1})^2 + \\
&\quad + \sum_{n=2}^{\infty}\frac{-1}{(n+1)}C^{\alpha\beta}_{11}(\sigma^\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\lambda^{(n-1)}_\nu A_\mu\ (u^{+1})^{n-1}(u^{-1})^{n+1}\,. && (3.12)
\end{aligned}
$$

Similarly, from eq. (3.4) we find

$$\delta^*_\Lambda D_{ij} = \begin{cases} 2\sqrt{2}C^{\mu\nu}_{11}\partial_\mu\chi\partial_\nu\bar{\phi}, & (i,j)=(1,1) \\ 0, & (i,j)\neq(1,1). \end{cases} \tag{3.13}$$

This comes from the fact that only the first term in the r.h.s. does not contain $u^+$ and it contributes to $\delta^*_\Lambda D_{11}$ because it is proportional to $(u^{-1})^2$. $\lambda^{(2,2)}$ is then determined as

$$\lambda^{(2,2)} = -i\sum_{n=2}^{\infty}\partial^\mu\lambda^{(n)}_\mu\ (u^{+1})^{n-2}(u^{-1})^{n+2}\,. \tag{3.14}$$

Eq. (3.5) is solved by

$$\delta^*_\Lambda\psi^i_\alpha = \delta^i_2\frac{2}{3}(\varepsilon C_{11}\sigma^\mu\bar{\psi}^1)_\alpha\partial_\mu\chi\,, \tag{3.15}$$

$$\begin{aligned} \lambda^{(1,2)}_\alpha &= \frac{4}{3}(\sigma^\mu\bar{\psi}_i)_\alpha C^{\gamma\delta}_{11}(\sigma_\mu\bar{\sigma}^\nu\varepsilon)_{\gamma\delta}\partial_\nu\chi\ u^{-(i}u^{-1}u^{-1)} + \\ &\quad + \sum_{n=2}^{\infty} i(\sigma^\mu\bar{\psi}_i)_\alpha C^{\gamma\delta}_{11}(\sigma_\mu\bar{\sigma}^\nu\varepsilon)_{\gamma\delta}\lambda^{(n-1)}_\nu \times \\ &\quad \times\left[a_n\,\overbrace{u^{+(1}\ldots u^{+1}}^{n-1}\overbrace{u^{-i}\ldots u^{-1)}}^{n+2} + b_n\epsilon^{i1}(u^{+1})^{n-2}(u^{-1})^{n+1}\right], \end{aligned} \tag{3.16}$$

with $a_n$ and $b_n$ being appropriate constants. Indeed, substituting (3.7) and (3.16) in appendix A into eq. (3.5) and using the formulas (A.1) and (A.2) we can see that only the first term in the r.h.s. of (3.5) gives a contribution to $\delta^*_\Lambda\psi^2$ that cannot be gauged away; the other terms are absorbed into $\lambda^{(1,2)}_\alpha$ (with the appropriate choice of $a_n$ and $b_n$).

As described above, the exact deformed gauge transformation is determined as

$$\begin{aligned} \delta^*_\Lambda A_\mu &= -\partial_\mu\chi\,, \\ \delta^*_\Lambda\psi^2_\alpha &= -\frac{2}{3}(\varepsilon C_{11}\sigma^\mu\bar{\psi}_2)_\alpha\partial_\mu\chi\,, \\ \delta^*_\Lambda D_{11} &= -2\sqrt{2}C^{\mu\nu}_{11}\partial_\mu(\partial_\nu\chi\bar{\phi})\,, \\ \delta^*_\Lambda(\text{others}) &= 0\,. \end{aligned} \tag{3.17}$$

Note that this result agrees with the deformed gauge transformation for the generic $C_{ij}$ in [14], after setting $C^{\alpha\beta}_{ij} = C^{\alpha\beta}_{11}\delta^1_i\delta^1_j$. For example, $\delta^*_\Lambda A_\mu$ was determined as $\delta^*_\Lambda A_\mu = -\{1+f(\bar{\phi})\bar{\phi}\}\partial_\mu\chi$ with $f(\bar{\phi})$ being a function proportional to $C_{12}$ or $C_{21}$ (see [14] for the definition of $f$), so that $f(\bar{\phi})=0$ when $C_{ij}=C_{11}\delta^1_i\delta^1_j$, which leads to eq. (3.6).

The determination of the exact $\mathcal{N}=(1,1/2)$ supersymmetry transformation is accomplished in a similar way as the one described above, since the deformation of the transformation laws comes from the associated gauge transformation to preserve the WZ gauge. We will give the result in the next subsection. The details of the derivation are found in appendix B.

### 3.2 Exact $\mathcal{N} = (1,\ 1/2)$ supersymmetry transformation

Since the theory is expected to have $\mathcal{N} = (1,\ 1/2)$ supersymmetry, we will concentrate on this symmetry in the following. For later convenience, we will split the $\mathcal{N} = (1,\ 1/2)$ supersymmetry transformation generated by $Q^1$, $Q^2$ and $\bar{Q}_2$ into the $\mathcal{N} = (1,0)$ transformation generated by $Q^1, Q^2$ and $\mathcal{N} = (0,1/2)$ by $\bar{Q}_2$. The $\mathcal{N} = (1,0)$ transformation is defined by

$$\delta^*_\xi V^{++}_{WZ} = \tilde{\delta}_\xi V^{++}_{WZ} + \delta^*_\Lambda V^{++}_{WZ}\,, \tag{3.18}$$

and the $\mathcal{N} = (0,1/2)$ transformation is

$$\delta^*_{\bar{\xi}^2} V^{++}_{WZ} = \tilde{\delta}_{\bar{\xi}^2} V^{++}_{WZ} + \delta^*_{\Lambda'} V^{++}_{WZ}\,. \tag{3.19}$$

Here

$$\tilde{\delta}_\xi V^{++}_{WZ} = \xi_i Q^i V^{++}_{WZ} = \left(-\xi^{+\alpha} Q^-_\alpha + \xi^{-\alpha} Q^+_\alpha\right) V^{++}_{WZ}\,, \tag{3.20}$$

$$\tilde{\delta}_{\bar{\xi}^2} V^{++}_{WZ} = \bar{\xi}^i \bar{Q}_i V^{++}_{WZ} = \left(\bar{\xi}^+_{\dot\alpha} \bar{Q}^{-\dot\alpha} - \bar{\xi}^-_{\dot\alpha} \bar{Q}^{+\dot\alpha}\right) V^{++}_{WZ} \tag{3.21}$$

and $Q^+_\alpha = \frac{\partial}{\partial\theta^{-\alpha}} - 2i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{+\dot\alpha}\frac{\partial}{\partial x^\mu_A}$, $Q^-_\alpha = -\frac{\partial}{\partial\theta^{+\alpha}}$, $\bar{Q}^+_{\dot\alpha} = \frac{\partial}{\partial\bar\theta^{-\dot\alpha}} + 2i\theta^{+\alpha}\sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial x^\mu_A}$, $\bar{Q}^-_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{+\dot\alpha}}$. Note that $\bar\xi^1$ is implicitly set to be zero in eq. (3.21), because we are now considering the $\mathcal{N} = (1,\ 1/2)$ supersymmetry. $\delta^*_\Lambda V^{++}_{WZ}$ ($\delta^*_{\Lambda'} V^{++}_{WZ}$) is a deformed gauge transformation of $V^{++}_{WZ}$ with an appropriate analytic gauge parameter $\Lambda(\zeta,u)$ ($\Lambda'(\zeta,u)$) to retain the WZ gauge.

The exact $\mathcal{N} = (1,0)$ transformation generated by $Q^1$ and $Q^2$ is given below (for the derivation, see appendix B.1):

$$\begin{aligned}
\delta^*_\xi \phi &= -\sqrt{2}i\xi^i\psi_i + \frac{8}{3}i(\xi_2\varepsilon C_{11}\psi^1)\bar\phi - \frac{2\sqrt{2}}{3}i(\xi_2\varepsilon C_{11}\sigma^\nu\bar\psi_2)A_\nu\,,\\
\delta^*_\xi \bar\phi &= 0\,,\\
\delta^*_\xi A_\mu &= i\xi^i\sigma_\mu\bar\psi_i + 2\sqrt{2}i(\xi^1\varepsilon C_{11}\sigma_\mu\bar\psi^1)\bar\phi\,,\\
\delta^*_\xi \psi^{\alpha i} &= -(\xi^i\sigma^{\mu\nu})^\alpha F_{\mu\nu} - D^{ij}\xi^\alpha_j + \sqrt{2}C^{\mu\nu}_{11}(\xi^{(i}\sigma_{\mu\nu})^\alpha D^{11)}\bar\phi - iC^{\mu\nu}_{11}(\xi^{(i}\sigma_{\mu\nu})^\alpha(\bar\psi^1\bar\psi^{1)}) +\\
&\quad + \delta^i_2\left\{2\sqrt{2}(\xi^1\varepsilon C_{11}\sigma^{\mu\nu})^\alpha + \frac{2\sqrt{2}}{3}(\xi^1\sigma^{\mu\nu}\varepsilon C_{11})^\alpha + \sqrt{2}C^{\mu\nu}_{11}\xi^{\alpha 1}\right\}\bar\phi F_{\mu\nu} -\\
&\quad - \delta^i_2\left\{\frac{4\sqrt{2}}{3}(\xi^1\sigma^{\mu\nu}\varepsilon C_{11})^\alpha + 2\sqrt{2}C^{\mu\nu}_{11}\xi^{\alpha 1}\right\}\partial_\mu\bar\phi A_\nu + \delta^i_2\frac{2\sqrt{2}}{3}(\xi^1\varepsilon C_{11})^\alpha\partial^\mu\bar\phi A_\mu +\\
&\quad + \delta^i_2 \det C_{11}\left\{\frac{8}{3}\xi^{\alpha 1}D^{11}\bar\phi^2 - \frac{8\sqrt{2}}{3}i\xi^{\alpha 1}(\bar\psi^1\bar\psi^1)\bar\phi\right\},\\
\delta^*_\xi \bar\psi_{\dot\alpha i} &= \sqrt{2}(\xi_i\sigma^\nu)_{\dot\alpha}\partial_\nu\bar\phi + 2\delta^1_i(\xi_2\varepsilon C_{11}\sigma^\nu)_{\dot\alpha}\partial_\nu(\bar\phi^2)\,,\\
\delta^*_\xi D^{11} &= -2i\xi^1\sigma^\nu\partial_\nu\bar\psi^1\,,\\
\delta^*_\xi D^{12} &= \delta^*_\xi D^{21} = -2i\xi^{(1}\sigma^\nu\partial_\nu\bar\psi^{2)} + 2\sqrt{2}i\xi^1\varepsilon C_{11}\sigma^\nu\partial_\nu(\bar\phi\bar\psi^1)\,,\\
\delta^*_\xi D^{22} &= -2i\xi^2\sigma^\nu\partial_\nu\bar\psi^2 + 4\sqrt{2}i\xi^{(1}\varepsilon C_{11}\sigma^\nu\partial_\nu(\bar\phi\bar\psi^{2)}) + 8i\det C_{11}\xi^1\sigma^\nu\partial_\nu(\bar\phi^2\bar\psi^1)\,.
\end{aligned} \tag{3.22}$$

Note that we have used $(\varepsilon C_{11}\varepsilon C_{11})_\alpha{}^\beta = -\delta^\beta_\alpha \det C_{11}$.

The $\mathcal{N}=(0,1/2)$ supersymmetry transformation generated by $\bar{Q}_2$ is as follows (these are read from eqs.(B.42)–(B.47) in appendix B.2 by setting $\bar{\xi}^1=0$) :

$$
\begin{aligned}
\delta^*_{\bar\xi^2}\phi &= 0\,,\\
\delta^*_{\bar\xi^2}\bar\phi &= \sqrt{2}i\bar\xi^2\bar\psi^1\,,\\
\delta^*_{\bar\xi^2}A_\mu &= -i\bar\xi^2\bar\sigma_\mu\psi^1\,,\\
\delta^*_{\bar\xi^2}\psi^{\alpha i} &= \delta^i_2\left\{-\sqrt{2}(\bar\xi^2\bar\sigma^\mu)^\alpha\partial_\mu\phi-\frac{2}{3}D^{11}(\bar\xi^2\bar\sigma^\mu\varepsilon C_{11})^\alpha A_\mu+\frac{4}{3}i\bar\xi^2\bar\psi^1(\psi^1\varepsilon C_{11})^\alpha\right\},\\
\delta^*_{\bar\xi^2}\bar\psi_{\dot\alpha i} &= \delta^1_i(\bar\xi^2\bar\sigma^{\mu\nu})_{\dot\alpha}F_{\mu\nu}-\bar\xi^2_{\dot\alpha}D_{i2}\,,\\
\delta^*_{\bar\xi^2}D^{11} &= 0\,,\\
\delta^*_{\bar\xi^2}D^{12} &= \delta^*_{\bar\xi^2}D^{21}=-i\partial_\mu(\bar\xi^2\bar\sigma^\mu\psi^1)\,,\\
\delta^*_{\bar\xi^2}D^{22} &= -2i\partial_\mu(\bar\xi^2\bar\sigma^\mu\psi^2)-\\
&\quad-\frac{2}{3}i\partial_\mu\left[2(\bar\xi^2\bar\sigma^\nu\varepsilon C_{11}\sigma^\mu\bar\psi^1)A_\nu-C^{\alpha\beta}_{11}(\sigma^\nu\bar\sigma^\mu\varepsilon)_{\alpha\beta}A_\nu\bar\xi^2\bar\psi^1+\sqrt{2}(\bar\xi^2\bar\sigma^\mu\varepsilon C_{11}\psi^1)\bar\phi\right].
\end{aligned}
\tag{3.23}
$$

From the exact gauge transformation (3.17), we easily find a field redefinition that leads to the canonical gauge transformation:

$$
\hat\psi^2_\alpha \equiv \psi^2_\alpha-\frac{2}{3}(\varepsilon C_{11}\sigma^\mu\bar\psi_2)_\alpha A_\mu\,,\tag{3.24}
$$

$$
\hat D_{11} \equiv D_{11}-2\sqrt{2}C^{\mu\nu}_{11}A_\nu\partial_\mu\bar\phi\,,\tag{3.25}
$$

and the other newly defined fields with hat are the same as the original fields.

Using this field redefinition, the exact $\mathcal{N}=(1,\ 1/2)$ transformation becomes as follows. The $\mathcal{N}=(1,0)$ supersymmetry transformation is

$$
\begin{aligned}
\delta^*_\xi\hat\phi &= -\sqrt{2}i\xi^i\hat\psi_i+\frac{8}{3}i(\xi_2\varepsilon C_{11}\hat\psi^1)\hat{\bar\phi}\,,\\
\delta^*_\xi\hat{\bar\phi} &= 0\,,\\
\delta^*_\xi\hat A_\mu &= i\xi^i\sigma_\mu\hat{\bar\psi}_i+2\sqrt{2}i(\xi^1\varepsilon C_{11}\sigma_\mu\hat{\bar\psi}^1)\hat{\bar\phi}\,,\\
\delta^*_\xi\hat\psi^{\alpha i} &= -(\xi^i\sigma^{\mu\nu})^\alpha\hat F_{\mu\nu}-\hat D^{ij}\xi^\alpha_j+\sqrt{2}C^{\mu\nu}_{11}(\xi^{(i}\sigma_{\mu\nu})^\alpha\hat D^{11)}\hat{\bar\phi}-iC^{\mu\nu}_{11}(\xi^i\sigma_{\mu\nu})^\alpha(\hat{\bar\psi}^1\hat{\bar\psi}^1)+\\
&\quad+\delta^i_2\left\{2\sqrt{2}(\xi^1\varepsilon C_{11}\sigma^{\mu\nu})^\alpha+\frac{2\sqrt{2}}{3}(\xi^1\sigma^{\mu\nu}\varepsilon C_{11})^\alpha+\sqrt{2}C^{\mu\nu}_{11}\xi^{\alpha 1}\right\}\hat{\bar\phi}\hat F_{\mu\nu}+\\
&\quad+\frac{8}{3}\delta^i_2\det C_{11}\xi^{\alpha 1}\left\{\hat D^{11}\hat{\bar\phi}^2-2\sqrt{2}i(\hat{\bar\psi}^1\hat{\bar\psi}^1)\hat{\bar\phi}\right\},\\
\delta^*_\xi\hat{\bar\psi}_{\dot\alpha i} &= \sqrt{2}(\xi_i\sigma^\nu)_{\dot\alpha}\partial_\nu\hat{\bar\phi}+2\delta^1_i(\xi_2\varepsilon C_{11}\sigma^\nu)_{\dot\alpha}\partial_\nu(\hat{\bar\phi}^2)\,,\\
\delta^*_\xi\hat D^{11} &= -2i\xi^1\sigma^\nu\partial_\nu\hat{\bar\psi}^1\,,\\
\delta^*_\xi\hat D^{12} &= \delta^*_\xi\hat D^{21}=-2i\xi^{(1}\sigma^\nu\partial_\nu\hat{\bar\psi}^{2)}+2\sqrt{2}i\xi^1\varepsilon C_{11}\sigma^\nu\partial_\nu(\hat{\bar\phi}\hat{\bar\psi}^1)\,,\\
\delta^*_\xi\hat D^{22} &= -2i\xi^2\sigma^\nu\partial_\nu\hat{\bar\psi}^2+4\sqrt{2}i\xi^{(1}\varepsilon C_{11}\sigma^\nu\partial_\nu(\hat{\bar\phi}\hat{\bar\psi}^{2)})-2\sqrt{2}i(\xi^i\varepsilon C_{11}\sigma^\nu\hat{\bar\psi}_i)\partial_\nu\hat{\bar\phi}+\\
&\quad+8i\det C_{11}\left\{\xi^1\sigma^\nu\partial_\nu(\hat{\bar\phi}^2\hat{\bar\psi}^1)+(\xi^1\sigma^\nu\hat{\bar\psi}^1)\partial_\nu\hat{\bar\phi}\hat{\bar\phi}\right\},
\end{aligned}
\tag{3.26}
$$

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where $\hat{F}_{\mu\nu} \equiv \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu$. The $\mathcal{N} = (0,\ 1/2)$ supersymmetry transformation becomes

$$\begin{aligned}
\delta^*_{\bar{\xi}^2} \hat{\phi} &= 0\,, \\
\delta^*_{\bar{\xi}^2} \hat{\bar{\phi}} &= \sqrt{2} i \bar{\xi}^2 \hat{\bar{\psi}}^1\,, \\
\delta^*_{\bar{\xi}^2} \hat{A}_\mu &= -i \bar{\xi}^2 \bar{\sigma}_\mu \hat{\psi}^1\,, \\
\delta^*_{\bar{\xi}^2} \hat{\psi}^{\alpha i} &= \delta^i_2 \left\{ -\sqrt{2} (\bar{\xi}^2 \bar{\sigma}^\mu)^\alpha \partial_\mu \hat{\phi} + \frac{8}{3} i \bar{\xi}^2 \hat{\bar{\psi}}^1 (\hat{\psi}^1 \varepsilon C_{11})^\alpha \right\}, \\
\delta^*_{\bar{\xi}^2} \hat{\bar{\psi}}_{\dot{\alpha} i} &= \delta^1_i (\bar{\xi}^2 \bar{\sigma}^{\mu\nu})_{\dot{\alpha}} \hat{F}_{\mu\nu} - \bar{\xi}^2_{\dot{\alpha}} \hat{D}_{i2}\,, \\
\delta^*_{\bar{\xi}^2} \hat{D}^{11} &= 0\,, \\
\delta^*_{\bar{\xi}^2} \hat{D}^{12} &= \delta^*_{\bar{\xi}^2} \hat{D}^{21} = -i \partial_\mu (\bar{\xi}^2 \bar{\sigma}^\mu \hat{\psi}^1)\,, \\
\delta^*_{\bar{\xi}^2} \hat{D}^{22} &= -2i \partial_\mu (\bar{\xi}^2 \bar{\sigma}^\mu \hat{\psi}^2) + 2i C^{\mu\nu}_{11} \bar{\xi}^2 \hat{\bar{\psi}}^1 \hat{F}_{\mu\nu} - \\
&\quad - 2\sqrt{2} i (\bar{\xi}^2 \bar{\sigma}^\mu \varepsilon C_{11} \hat{\psi}^1) \partial_\mu \hat{\bar{\phi}} - \frac{2\sqrt{2}}{3} i \partial_\mu \left[ (\bar{\xi}^2 \bar{\sigma}^\mu \varepsilon C_{11} \hat{\psi}^1) \hat{\bar{\phi}} \right].
\end{aligned} \tag{3.27}$$

In terms of the newly defined fields, the action in the WZ gauge should now be invariant under the ordinary gauge transformation and the above $\mathcal{N} = (1,\ 1/2)$ supersymmetry transformation.

Note that the $O(C)$ action is exactly gauge and $\mathcal{N} = (1,\ 1/2)$ supersymmetry invariant by itself (in order to show this we have used $(\bar{\psi}^1)^3 = 0$ as a result of the U(1) gauge group). An immediate consequence of this fact is that the terms arising from $(V^{++})^n$ $(n \geq 4)$ in the action, if there exist, have to be also gauge and $\mathcal{N} = (1,\ 1/2)$ supersymmetry invariant by themselves.

Using the field redefinitions (2.25) and (2.26) and transformation (3.27), we find that the action (2.5) is invariant under the $\mathcal{N} = (0, 1/2)$ transformation:

$$\begin{aligned}
\delta^*_{\bar{\eta}} A &= 0\,, \qquad \delta^*_{\bar{\eta}} \bar{A} = \sqrt{2} \bar{\eta} \bar{\lambda}\,, \\
\delta^*_{\bar{\eta}} \psi^\alpha &= 0\,, \qquad \delta^*_{\bar{\eta}} \bar{\psi}_{\dot{\alpha}} = (\bar{\eta} \bar{\sigma}^{\mu\nu})_{\dot{\alpha}} v_{\mu\nu} - i \bar{\eta}_{\dot{\alpha}} D\,, \\
\delta^*_{\bar{\eta}} F &= 0\,, \qquad \delta^*_{\bar{\eta}} \bar{F} = \sqrt{2} i \bar{\eta} \bar{\sigma}^\mu \partial_\mu \lambda + 2 \bar{\eta} \bar{\sigma}^\mu \varepsilon C \partial_\mu (\psi \bar{A})\,, \\
\delta^*_{\bar{\eta}} v_\mu &= -i \bar{\eta} \bar{\sigma}_\mu \psi\,, \\
\delta^*_{\bar{\eta}} \lambda^\alpha &= -i \sqrt{2} (\bar{\eta} \bar{\sigma}^\mu)^\alpha \partial_\mu A - 2i \bar{\eta} \bar{\lambda} (\psi \varepsilon C)^\alpha\,, \qquad \delta^*_{\bar{\eta}} \bar{\lambda}_{\dot{\alpha}} = \sqrt{2} \bar{\eta}_{\dot{\alpha}} F\,, \\
\delta^*_{\bar{\eta}} D &= \bar{\eta} \bar{\sigma}^\mu \partial_\mu \psi\,.
\end{aligned} \tag{3.28}$$

Thus we have obtained deformed $\mathcal{N} = (1,\ 1/2)$ supersymmetry transformations (2.8), (2.27) and (3.28) in $\mathcal{N} = 2$ supersymmetric U(1) gauge theory in the deformed $\mathcal{N} = 1$ superspace.

## 4. Conclusions and discussion

In this paper, we studied $\mathcal{N} = 2$ supersymmetric U(1) gauge theory in non(anti)commutative $\mathcal{N} = 1$ and $\mathcal{N} = 2$ harmonic superspaces. We considered the reduction of the deformation parameters in non(anti)commutative harmonic superspace to the deformed $\mathcal{N} = 1$ superspace. We found that the $O(C)$ action in harmonic superspace reduces to the one in the deformed $\mathcal{N} = 1$ superspace by the field redefinition. We calculated the gauge and $\mathcal{N} = (1, 1/2)$ supersymmetry transformations exactly. Using the field redefinition, we confirmed $\mathcal{N} = (1, 1/2)$ supersymmetry of the action in the deformed $\mathcal{N} = 1$ superspace.

It is known that even in the U(1) gauge group there exists higher order $C$-corrections to the action in the deformed harmonic superspace [14, 15]. It is not clear whether these higher order contributions in the action disappear after the reduction of the deformation parameters (2.23). Even if these terms exist after the reduction, these must be gauge- and $\mathcal{N} = (1, 1/2)$ supersymmetry invariant by themselves and reduce to the deformed action in the $\mathcal{N} = 1$ superspace formalism by appropriate field redefinition. Some detailed analysis will be studied elsewhere.

In this paper, the component $\mathcal{N} = (1, 1/2)$ supersymmetry transformation for the action in the deformed $\mathcal{N} = 1$ superspace has not fixed completely, because it is unclear whether the field identification (2.25) is exact or not. To fix the component transformation, it is useful to work with the $\mathcal{N} = 2$ rigid superspace formalism. The deformed action could be written in terms of the $\mathcal{N} = 2$ chiral superfield that describes the $\mathcal{N} = 2$ vector multiplet and give directly the action in the deformed $\mathcal{N} = 1$ superspace. If we can construct the appropriately deformed $\mathcal{N} = 2$ chiral superfield, the component transformation that is not fixed in this paper will be determined. This formalism would be also useful to study a generalization to non-abelian gauge group.

Another interesting issue would be a central extension of $\mathcal{N} = (1, 1/2)$ supersymmetry algebra. It is also interesting to study properties of deformed properties of solitons in such as monopoles and instantons in such theories [9].

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## A. Useful formulas

In this appendix, we describe some useful reduction formulas of harmonic variables, which we have used in section 3 to determine the exact transformation laws in the harmonic superspace formalism. They are

$$
\begin{aligned}
u^{+(1}u^{+1)}\,&\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-j}u^{-k}u^{-1}\cdots u^{-1)}}^{m} = \\
&= \overbrace{u^{+(1}\cdots u^{+1}}^{n+2}\overbrace{u^{-i}u^{-j}u^{-k}u^{-1}\cdots u^{-1)}}^{m} + \\
&+ \frac{2m}{(n+m+2)(n+m)} \times \\
&\times \Big(\epsilon^{i1}\,\overbrace{u^{+(1}\cdots u^{+1}}^{n+1}\overbrace{u^{-j}u^{-k}u^{-1}\cdots u^{-1)}}^{m-1} + (\text{cyclic permut. of } (i,j,k))\Big) +
\end{aligned}
$$

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$$
\begin{aligned}
&+\frac{2m(m-1)}{(n+m+1)(n+m)^2(n+m-1)}\times\\
&\times\Big(\epsilon^{i1}\epsilon^{j1}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-k}u^{-1}\cdots u^{-1)}}^{m-2}+(\text{cyclic permut. of }(i,j,k))\Big)\,,
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
&u^{-k}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-j}u^{-1}\cdots u^{-1)}}^{m}=\\
&=\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-j}u^{-k}u^{-1}\cdots u^{-1)}}^{m+1}+\\
&+\frac{2n}{(n+m+1)(n+m)}\frac{1}{2}\Big\{\epsilon^{ki}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-j}u^{-1}\cdots u^{-1)}}^{m}+(i\leftrightarrow j)\Big\}+\\
&+\frac{n(n+m-2)}{(n+m+1)(n+m)}\epsilon^{k1}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-i}u^{-j}u^{-1}\cdots u^{-1)}}^{m}\,,
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
&u^{-(k}u^{-l)}\overbrace{u^{+(i}u^{+1}\cdots u^{+1}}^{n}\overbrace{u^{-1}\cdots u^{-1)}}^{m}=\\
&=\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-k}u^{-l}u^{-1}\cdots u^{-1)}}^{m+2}+\\
&+\frac{2n(n+m-1)}{(n+m+2)(n+m)}\frac{1}{2}\Big\{\epsilon^{k1}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-l}u^{-i}u^{-1}\cdots u^{-1)}}^{m+1}+(k\leftrightarrow l)\Big\}+\\
&+\frac{2n}{(n+m+2)(n+m)}\frac{1}{2}\Big\{\epsilon^{ki}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-l}u^{-1}\cdots u^{-1)}}^{m+1}+(k\leftrightarrow l)\Big\}+\\
&+\frac{n(n-1)(n+m-2)}{(n+m+1)(n+m)^2}\epsilon^{l1}\epsilon^{k1}\overbrace{u^{+(1}\cdots u^{+1}}^{n-2}\overbrace{u^{-i}u^{-1}\cdots u^{-1)}}^{m}+\\
&+\frac{2n(n-1)}{(n+m+1)(n+m)^2}\frac{1}{2}\Big\{\epsilon^{l1}\epsilon^{ki}\overbrace{u^{+(1}\cdots u^{+1}}^{n-2}\overbrace{u^{-1}\cdots u^{-1)}}^{m}+(k\leftrightarrow l)\Big\}\,,
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
&u^{+(1}u^{-1)}\overbrace{u^{+(i}u^{+1}\ldots u^{+1}}^{n}\overbrace{u^{-k}u^{-1}\ldots u^{-1)}}^{m}=\\
&=\overbrace{u^{+(i}u^{+1}\ldots u^{+1}}^{n+1}\overbrace{u^{-k}u^{-1}\ldots u^{-1)}}^{m+1}-\\
&-\frac{2(n-m)}{(n+m+2)(n+m)}\frac{1}{2}\Big(\epsilon^{i1}\overbrace{u^{+(1}\ldots u^{+1}}^{n}\overbrace{u^{-k}u^{-1}\ldots u^{-1)}}^{m}+(i\leftrightarrow k)\Big)-\\
&-\frac{2nm}{(n+m+1)(n+m)^2(n+m-1)}\epsilon^{i1}\epsilon^{k1}\overbrace{u^{+(1}\ldots u^{+1}}^{n-1}\overbrace{u^{-1}\ldots u^{-1)}}^{m-1}\,.
\end{aligned}
\tag{A.4}
$$

## B. $\mathcal{N}=(1,\,1/2)$ supersymmetry

Here we will present the details of calculation of the exact $\mathcal{N}=(1,\,1/2)$ supersymmetry transformation of the component fields when the deformation parameters are restricted as (2.23). The result has been summarized in section 3.2.

### B.1 $\mathcal{N}=(1,0)$ supersymmetry

The equations to determine the deformed $\mathcal{N}=(1,0)$ supersymmetry transformation are eqs.(B.1)–(B.8) which are obtained from eq.(3.18) and (3.1):

$$
\begin{aligned}
0 &= 2i(\xi^+\sigma^\mu)_{\dot\beta}\varepsilon^{\dot\beta\dot\alpha}A_\mu - \partial^{++}\lambda^{(0,1)\dot\alpha} - 2\lambda^{(1,0)\alpha}(\varepsilon C^{++}\sigma^\mu)_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}A_\mu\,, && \text{(B.1)}\\
0 &= -2\sqrt{2}i\xi^+_\alpha\bar\phi - \partial^{++}\lambda^{(1,0)}_\alpha - 2\sqrt{2}(\varepsilon C^{++}\lambda^{(1,0)})_\alpha\bar\phi\,, && \text{(B.2)}\\
\sqrt{2}i\delta^*_\xi\phi &= 4\xi^+\psi^i u^-_i + 4i\lambda^{(1,0)\alpha}(\varepsilon C^{++}\psi^i)_\alpha u^-_i - \\
&\quad - \partial^{++}\lambda^{(0,2)} - C^{++\alpha\beta}(\sigma^\nu\bar\sigma^\mu\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_\mu A_\nu\,, && \text{(B.3)}\\
-\sqrt{2}i\delta^*_\xi\bar\phi &= 0\,, && \text{(B.4)}\\
-2i\delta^*_\xi A_\mu &= 4\xi^+\sigma_\mu\bar\psi^i u^-_i + 4i\lambda^{(1,0)\alpha}(\varepsilon C^{++}\sigma_\mu\bar\psi^i)_\alpha u^-_i - \\
&\quad - \partial^{++}\lambda^{(1,1)}_\mu - \sqrt{2}C^{++\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_\nu\bar\phi\,, && \text{(B.5)}\\
4\delta^*_\xi\psi^i_\alpha u^-_i &= -2(\sigma^\mu\bar\sigma^\nu\xi^-)_\alpha\partial_\nu A_\mu + 6\xi^+_\alpha D^{ij}u^-_i u^-_j - \\
&\quad - i(\sigma^\nu\partial_\nu\lambda^{(0,1)})_\alpha - 2\sqrt{2}i(\varepsilon C^{+-}\sigma^\nu\partial_\nu\lambda^{(0,1)})_\alpha\bar\phi - \\
&\quad - 6i(\varepsilon C^{++}\lambda^{(1,0)})_\alpha D^{ij}u^-_i u^-_j + 2i\lambda^{(1,0)\beta}(\varepsilon C^{+-}\sigma^\nu\bar\sigma^\mu\varepsilon)_{\beta\alpha}\partial_\nu A_\mu + \\
&\quad + 2i\partial_\nu\lambda^{(1,0)}_\alpha C^{+-\beta\gamma}(\sigma^\mu\bar\sigma^\nu\varepsilon)_{\beta\gamma}A_\mu - 2i(\sigma^\mu\bar\psi^i)_\alpha u^-_i C^{++\gamma\delta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\gamma\delta}\lambda^{(1,1)}_\nu - \\
&\quad - \partial^{++}\lambda^{(1,2)}_\alpha - 2\sqrt{2}(\varepsilon C^{++}\lambda^{(1,2)})_\alpha\bar\phi - 2(\varepsilon C^{++}\sigma^\mu\lambda^{(2,1)})_\alpha A_\mu\,, && \text{(B.6)}\\
-4\delta^*_\xi\bar\psi^{\dot\alpha i}u^-_i &= 2\sqrt{2}(\xi^-\sigma^\mu)_{\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\partial_\mu\bar\phi + i\partial_\mu\lambda^{(1,0)\alpha}\sigma^\mu_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha} + \\
&\quad + 2\sqrt{2}i\partial_\nu\left\{\lambda^{(1,0)\alpha}(\varepsilon C^{+-}\sigma^\nu)_{\alpha\dot\beta}\varepsilon^{\dot\beta\dot\alpha}\bar\phi\right\} - \partial^{++}\lambda^{(2,1)\dot\alpha}\,, && \text{(B.7)}\\
3\delta^*_\xi D^{ij}u^-_i u^-_j &= -4i\xi^-\sigma^\mu\partial_\mu\bar\psi^i u^-_i + 4\partial_\nu\left\{\lambda^{(1,0)\alpha}(\varepsilon C^{+-}\sigma^\nu\bar\psi^i)_\alpha u^-_i\right\} - \\
&\quad - i\partial^\mu\lambda^{(1,1)}_\mu - \sqrt{2}iC^{+-\alpha\beta}(\sigma^\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\partial_\nu(\lambda^{(1,1)}_\mu\bar\phi) - \partial^{++}\lambda^{(2,2)}\,. && \text{(B.8)}
\end{aligned}
$$

Note that under the restriction (2.23), we have $C^{+-\alpha\beta}=C^{\alpha\beta}_{11}u^{+1}u^{-1}$ and so on.

Eq. (B.2) is solved by

$$
\begin{aligned}
\lambda^{(1,0)\alpha} &= 2\sqrt{2}i\xi^\alpha_i\bar\phi\; u^{-i} + \sum_{n=1}^{\infty} 2\sqrt{2}i\xi^{(n)\alpha}_i\bar\phi \times \\
&\quad \times\left[\alpha^{(1,0)}_n \overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-1}\cdots u^{-1)}}^{n+1} + \beta^{(1,0)}_n\epsilon^{i1}(u^{+1})^{n-1}(u^{-1})^n\right], \qquad \text{(B.9)}
\end{aligned}
$$

where

$$
\xi^{(n)\alpha}_i \equiv (\xi_i\overbrace{\varepsilon C_{11}\cdots\varepsilon C_{11}}^{n})^\alpha(2\sqrt{2}\bar\phi)^n \qquad \text{(B.10)}
$$

and

$$
\begin{aligned}
(\alpha^{(1,0)}_1,\beta^{(1,0)}_1) &= \left(\frac{1}{2},\frac{2}{3}\right),\\
\alpha^{(1,0)}_n &= \frac{1}{n+1}\alpha^{(1,0)}_{n-1}\,,\quad \beta^{(1,0)}_n = \frac{1}{n}\left(\frac{2n}{(2n+1)(2n-1)}\alpha^{(1,0)}_{n-1} + \beta^{(1,0)}_{n-1}\right). \qquad \text{(B.11)}
\end{aligned}
$$

We then obtain $\alpha^{(1,0)}_n = \frac{1}{(n+1)!}$, $\beta^{(1,0)}_n = \frac{2}{(n-1)!(2n+1)}$. To check (B.11), we need (A.1), (A.2) and

$$
2\sqrt{2}(\xi^{(n)}_i\varepsilon C_{11})^\alpha\bar\phi = \xi^{(n+1)\alpha}_i\,. \qquad \text{(B.12)}
$$

From eq. (B.1) we find

$$\lambda^{(0,1)}_{\dot\alpha} = 2i(\xi_i\sigma^\mu)_{\dot\alpha}A_\mu\; u^{-i} + \tag{B.13}$$
$$+\sum_{n=1}^{\infty} 2i(\xi_i^{(n)}\sigma^\mu)_{\dot\alpha}A_\mu\left[\alpha_n^{(1,0)}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-1}\cdots u^{-1)}}^{n+1}+\beta_n^{(1,0)}\epsilon^{i1}(u^{+1})^{n-1}(u^{-1})^n\right].$$

In eq. (B.5), $\lambda^{(1,1)}_\mu$ is given by

$$\begin{aligned}\lambda^{(1,1)}_\mu =\;& 2\xi_i\sigma_\mu\bar\psi_j\; u^{-(i}u^{-j)} + \\ &+\xi_i^{(1)}\sigma_\mu\bar\psi_j\left[2u^{+(1}u^{-i}u^{-j}u^{-1)}+\frac{3}{2}\left\{\epsilon^{i1}u^{-(j}u^{-1)}+(i\leftrightarrow j)\right\}\right]+\\ &+\sum_{n=2}^{\infty}\xi_i^{(n)}\sigma_\mu\bar\psi_j\left[\alpha_n^{(1,1)}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-j}u^{-1}\cdots u^{-1)}}^{n+2}+\right.\\ &\qquad+\beta_n^{(1,1)}\frac{1}{2}\left\{\epsilon^{i1}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-j}u^{-1}\cdots u^{-1)}}^{n+1}+(i\leftrightarrow j)\;\right\}+\\ &\qquad\left.+\gamma_n^{(1,1)}\epsilon^{i1}\epsilon^{j1}(u^{+1})^{n-2}(u^{-1})^n+\delta_n^{(1,1)}\epsilon^{ji}(u^{+1})^{n-1}(u^{-1})^{n+1}\right],\end{aligned}\tag{B.14}$$

where $(\alpha_2^{(1,1)},\beta_2^{(1,1)},\gamma_2^{(1,1)},\delta_2^{(1,1)})=(1,\frac{8}{3},\frac{8}{5},-\frac{1}{3})$ and

$$\begin{aligned}\alpha_n^{(1,1)} &= \frac{1}{n+1}\left(4\alpha_{n-1}^{(1,0)}+\alpha_{n-1}^{(1,1)}\right),\\ \beta_n^{(1,1)} &= \frac{1}{n+1}\left(\frac{4n}{2n-1}\alpha_{n-1}^{(1,0)}+4\beta_{n-1}^{(1,0)}+\frac{1}{n}\alpha_{n-1}^{(1,1)}+\beta_{n-1}^{(1,1)}\right),\\ \gamma_n^{(1,1)} &= \frac{1}{n}\left(\frac{4n}{(2n+1)(2n-1)}\alpha_{n-1}^{(1,0)}+2\beta_{n-1}^{(1,0)}+\right.\\ &\qquad\left.+\frac{n+1}{(2n+1)2n(2n-1)}\alpha_{n-1}^{(1,1)}+\frac{1}{2n-2}\beta_{n-1}^{(1,1)}+\gamma_{n-1}^{(1,1)}\right),\\ \delta_n^{(1,1)} &= \frac{1}{n+1}\left(\frac{2(n-1)}{2n-1}\alpha_{n-1}^{(1,0)}-2\beta_{n-1}^{(1,0)}+\delta_{n-1}^{(1,1)}\right).\end{aligned}\tag{B.15}$$

Substituting eq. (B.14) to the r.h.s. of eq. (B.5) and using (A.1), (A.2) and

$$-\sqrt{2}C^{++\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}(\xi_i^{(n)}\sigma_\nu\bar\psi_j)\bar\phi=(\xi_i^{(n+1)}\sigma_\mu\bar\psi_j)(u^{+1})^2\,,\tag{B.16}$$

we can see that all the $O(C^2)$ terms are gauged away and the contributions to $\delta^*_\xi A_\mu$ come only from the first term and the order $C$ part of the second term in (B.5).

Eq. (B.3) have the same structure as eq. (B.5) in terms of the harmonic variables. Therefore, it is solved in a way similar to (B.5): The $O(C^2)$ terms in the r.h.s. of eq. (B.3) are completely gauged away, so that we need to consider only the first and the second term to determine $\delta^*_\xi\phi$.

In eq. (B.7), $\lambda^{(2,1)}_{\dot\alpha}$ is given by

$$\lambda^{(2,1)}_{\dot\alpha} = \sqrt{2}\partial_\mu\left\{(\xi_i^{(1)}\sigma^\mu)_{\dot\alpha}\bar\phi\right\}u^{-(i}u^{-1}u^{-1)}+$$

$$+\sum_{n=2}^{\infty} 2\sqrt{2}\partial_\mu \left\{(\xi_i^{(n)}\sigma^\mu)_{\dot\alpha}\bar\phi\right\}\left[\alpha_n^{(2,1)}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-i}u^{-1}\cdots u^{-1)}}^{n+2}+\right.$$
$$\left.+\beta_n^{(2,1)}\epsilon^{i1}(u^{+1})^{n-2}(u^{-1})^{n+1}\right], \tag{B.17}$$

where

$$\alpha_n^{(2,1)}=\frac{1}{n+2}\left(\alpha_n^{(1,0)}+\alpha_{n-1}^{(1,0)}\right),\quad \beta_n^{(2,1)}=\frac{1}{n+1}\left(\frac{1}{(2n+1)(2n-1)}\alpha_{n-1}^{(1,0)}+\beta_n^{(1,0)}+\beta_{n-1}^{(1,0)}\right). \tag{B.18}$$

We can easily find that in the r.h.s. of (B.7) the terms proportional to $(C_{11})^n$ $(n\geq 2)$ are completely cancelled by this gauge parameter, so that there are no $O((C_{11})^2)$ terms in $\delta_\xi^*\bar\psi^i$.

Substituting (B.13), (B.9), (B.14) and (B.17) to eq. (B.6), we can check that the following form of the gauge parameter is sufficient :

$$\begin{aligned}
\lambda_\alpha^{(1,2)} = &\sum_{n=0}^{\infty}\chi_{\alpha ijk(n)}\overbrace{u^{+(1}\ldots u^{+1}}^{n}\overbrace{u^{-i}u^{-j}u^{-k}u^{-1}\ldots u^{-1)}}^{n+3}+\\
&+\sum_{n=1}^{\infty}\chi_{\alpha ij(n)}\overbrace{u^{+(1}\ldots u^{+1}}^{n-1}\overbrace{u^{-i}u^{-j}u^{-1}\ldots u^{-1)}}^{n+2}\\
&+\sum_{n=1}^{\infty}\chi_{\alpha i(n)}\overbrace{u^{+(1}\ldots u^{+1}}^{n-1}\overbrace{u^{-i}u^{-1}\ldots u^{-1)}}^{n+2}+\sum_{n=2}^{\infty}\tilde\chi_{\alpha i(n)}\overbrace{u^{+(1}\ldots u^{+1}}^{n-2}\overbrace{u^{-i}u^{-1}\ldots u^{-1)}}^{n+1}+\\
&+\sum_{n=2}^{\infty}\chi_{\alpha(n)}\overbrace{u^{+(1}\ldots u^{+1}}^{n-2}\overbrace{u^{-1}\ldots u^{-1)}}^{n+1}+\sum_{n=3}^{\infty}\tilde\chi_{\alpha(n)}\overbrace{u^{+(1}\ldots u^{+1}}^{n-3}\overbrace{u^{-1}\ldots u^{-1)}}^{n},
\end{aligned} \tag{B.19}$$

where the subscript $n$ denotes the power of $C_{11}$-dependence of each quantity. To see this is sufficient, we need (A.1)–(A.4). Note that in the r.h.s. of eq. (B.6) the terms proportional to $(C_{11})^n$ $(n\geq 3)$ are completely gauged away, so that there are no $O((C_{11})^3)$ terms in $\delta_\xi^*\psi^i$. In fact, in order to determine $\delta_\xi^*\psi^i$, only the precise forms of $\chi^\alpha_{ijk(0)}$ and $\chi^\alpha_{ij(1)}$ are needed:

$$\chi^\alpha_{ijk(0)} = 2\xi^\alpha_{(i}D_{jk)}, \tag{B.20}$$
$$\chi^\alpha_{ij(1)} = \frac{32}{15}\left[\left\{i(\bar\psi_i\bar\psi_j)-\sqrt{2}D_{ij}\bar\phi\right\}(\xi_k\varepsilon C_{11})^\alpha+\Big(\text{cyclic permut. of }(ijk)\Big)\right]\epsilon^{k1} \tag{B.21}$$

Substituting (B.9) and (B.14) to eq. (B.8) and collecting $u^+$-independent terms, we obtain $\delta_\xi^* D^{ij}$. We do not need the precise form of $\lambda^{(2,2)}$. To determine $\delta_\xi^* D^{ij}$, we can use (A.4) and (A.2).

### B.2 $\mathcal{N}=(0,1/2)$ supersymmetry

The equations to determine the deformed $\mathcal{N}=(0,1/2)$ supersymmetry transformation are eqs. (B.22)–(B.29) which are obtained from eq. (3.19) (note that here the expression (3.1) is used for the analytic gauge parameter $\Lambda'$):

$$
\begin{aligned}
0 &= 2\sqrt{2}i\bar{\xi}^{+\dot{\alpha}}\phi - \partial^{++}\lambda^{(0,1)\dot{\alpha}} - 2\lambda^{(1,0)\alpha}(\varepsilon C^{++}\sigma^{\mu})_{\alpha\dot{\beta}}\varepsilon^{\dot{\beta}\dot{\alpha}}A_{\mu}\,, && \text{(B.22)}\\
0 &= -2i(\sigma^{\mu}\bar{\xi}^{+})_{\alpha}A_{\mu} - \partial^{++}\lambda^{(1,0)}_{\alpha} - 2\sqrt{2}(\varepsilon C^{++}\lambda^{(1,0)})_{\alpha}\bar{\phi}\,, && \text{(B.23)}\\
\sqrt{2}i\delta^{*}_{\xi}\phi &= 4i\lambda^{(1,0)\alpha}(\varepsilon C^{++}\psi^{i})_{\alpha}u^{-}_{i} + \\
&\quad + \frac{\sqrt{2}}{3}\partial_{\mu}\lambda^{(2,0)}\partial_{\nu}\bar{\phi}\; C^{\alpha_1\beta_1}_{i_1j_1}C^{\alpha_2\beta_2}_{i_2j_2}C^{\alpha_3\beta_3}_{i_3j_3}\times\\
&\quad \times(-u^{-i_1}u^{+i_2}u^{+i_3}\varepsilon_{\alpha_3\alpha_2}\sigma^{\mu}_{\alpha_1\dot{\alpha}} + u^{+i_1}u^{-i_2}u^{+i_3}\varepsilon_{\alpha_3\alpha_1}\sigma^{\mu}_{\alpha_2\dot{\alpha}} - u^{+i_1}u^{+i_2}u^{-i_3}\varepsilon_{\alpha_1\alpha_2}\sigma^{\mu}_{\alpha_3\dot{\alpha}})\times\\
&\quad \times(-u^{-j_1}u^{+j_2}u^{+j_3}\varepsilon_{\beta_3\beta_2}\sigma^{\nu}_{\beta_1\dot{\beta}} + u^{+j_1}u^{-j_2}u^{+j_3}\varepsilon_{\beta_3\beta_1}\sigma^{\nu}_{\beta_2\dot{\beta}} - u^{+j_1}u^{+j_2}u^{-j_3}\varepsilon_{\beta_1\beta_2}\sigma^{\nu}_{\beta_3\dot{\beta}}) - \\
&\quad - \partial^{++}\lambda^{(0,2)} - C^{++\alpha\beta}(\sigma^{\nu}\bar{\sigma}^{\mu}\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_{\mu}A_{\nu}\,, && \text{(B.24)}\\
-\sqrt{2}i\delta^{*}_{\xi}\bar{\phi} &= -4\bar{\xi}^{+}\bar{\psi}^{i}u^{-}_{i} - \partial^{++}\lambda^{(2,0)}\,, && \text{(B.25)}\\
-2i\delta^{*}_{\xi}A_{\mu} &= -4\psi^{i}\sigma_{\mu}\bar{\xi}^{+}u^{-}_{i} + 4i\lambda^{(1,0)\alpha}(\varepsilon C^{++}\sigma_{\mu}\bar{\psi}^{i})_{\alpha}u^{-}_{i} - 2C^{++\alpha\beta}(\sigma^{\nu}\bar{\sigma}_{\mu}\varepsilon)_{\alpha\beta}A_{\nu}\lambda^{(2,0)} - \\
&\quad - \partial^{++}\lambda^{(1,1)}_{\mu} - \sqrt{2}C^{++\alpha\beta}(\sigma_{\mu}\bar{\sigma}^{\nu}\varepsilon)_{\alpha\beta}\lambda^{(1,1)}_{\nu}\bar{\phi}\,, && \text{(B.26)}\\
4\delta^{*}_{\xi}\psi^{i}_{\alpha}u^{-}_{i} &= 2\sqrt{2}(\sigma^{\mu}\bar{\xi}^{-})_{\alpha}\partial_{\mu}\phi - \\
&\quad - i(\sigma^{\nu}\partial_{\nu}\lambda^{(0,1)})_{\alpha} - 2\sqrt{2}i(\varepsilon C^{+-}\sigma^{\nu}\partial_{\nu}\lambda^{(0,1)})_{\alpha}\bar{\phi} - \\
&\quad - 6i(\varepsilon C^{++}\lambda^{(1,0)})_{\alpha}D^{ij}u^{-}_{i}u^{-}_{j} + 2i\lambda^{(1,0)\beta}(\varepsilon C^{+-}\sigma^{\nu}\bar{\sigma}^{\mu}\varepsilon)_{\beta\alpha}\partial_{\nu}A_{\mu} + \\
&\quad + 2i\partial_{\nu}\lambda^{(1,0)}_{\alpha}C^{+-\beta\gamma}(\sigma^{\mu}\bar{\sigma}^{\nu}\varepsilon)_{\beta\gamma}A_{\mu} + 8i(\varepsilon C^{++}\psi^{-})_{\alpha}\lambda^{(2,0)} - \\
&\quad - 2i(\sigma^{\mu}\bar{\psi}^{i})_{\alpha}u^{-}_{i}C^{++\gamma\delta}(\sigma_{\mu}\bar{\sigma}^{\nu}\varepsilon)_{\gamma\delta}\lambda^{(1,1)}_{\nu} - \\
&\quad - \partial^{++}\lambda^{(1,2)}_{\alpha} - 2\sqrt{2}(\varepsilon C^{++}\lambda^{(1,2)})_{\alpha}\bar{\phi} - 2(\varepsilon C^{++}\sigma^{\mu}\lambda^{(2,1)})_{\alpha}A_{\mu}\,, && \text{(B.27)}\\
-4\delta^{*}_{\xi}\bar{\psi}^{\dot{\alpha}i}u^{-}_{i} &= 6\bar{\xi}^{+\dot{\alpha}}D^{ij}u^{-}_{i}u^{-}_{j} - 2(\bar{\sigma}^{\nu}\sigma^{\mu}\bar{\xi}^{-})^{\dot{\alpha}}\partial_{\mu}A_{\nu} + i\partial_{\mu}\lambda^{(1,0)\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\varepsilon^{\dot{\beta}\dot{\alpha}} + \\
&\quad + 2\sqrt{2}i\partial_{\nu}\left\{\lambda^{(1,0)\alpha}(\varepsilon C^{+-}\sigma^{\nu})_{\alpha\dot{\beta}}\varepsilon^{\dot{\beta}\dot{\alpha}}\bar{\phi}\right\} - \partial^{++}\lambda^{(2,1)\dot{\alpha}}\,, && \text{(B.28)}\\
3\delta^{*}_{\xi}D^{ij}u^{-}_{i}u^{-}_{j} &= -4i\bar{\xi}^{-}\bar{\sigma}^{\mu}\partial_{\mu}\psi^{i}u^{-}_{i} + 4\partial_{\nu}\left\{\lambda^{(1,0)\alpha}(\varepsilon C^{+-}\sigma^{\nu}\bar{\psi}^{i})_{\alpha}u^{-}_{i}\right\} + \\
&\quad + 2iC^{+-\alpha\beta}(\sigma^{\mu}\bar{\sigma}^{\nu}\varepsilon)_{\alpha\beta}\partial_{\nu}(\lambda^{(2,0)}A_{\mu}) - \\
&\quad - i\partial^{\mu}\lambda^{(1,1)}_{\mu} - \sqrt{2}iC^{+-\alpha\beta}(\sigma^{\mu}\bar{\sigma}^{\nu}\varepsilon)_{\alpha\beta}\partial_{\nu}(\lambda^{(1,1)}_{\mu}\bar{\phi}) - \partial^{++}\lambda^{(2,2)}\,. && \text{(B.29)}
\end{aligned}
$$

Here we should understand that $\bar{\xi}^{1}$ is implicitly set to be zero. Note that under the restriction (2.23), we have $C^{+-\alpha\beta} = C^{\alpha\beta}_{11}u^{+1}u^{-1}$ and so on.

Eq.(B.23) is solved by

$$
\begin{aligned}
\lambda^{(1,0)\alpha} &= -2i(\bar{\xi}_{i}\bar{\sigma}^{\mu})^{\alpha}A_{\mu}\;u^{-i} + \sum_{n=1}^{\infty}(-2i)(\bar{\xi}_{i}\bar{\sigma}^{\mu}\varepsilon\mathcal{C}^{(n)})^{\alpha}A_{\mu}(2\sqrt{2}\bar{\phi})^{n}\times\\
&\quad \times\left[\alpha^{(1,0)}_{n}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-1}\cdots u^{-1)}}^{n+1} + \beta^{(1,0)}_{n}\epsilon^{i1}(u^{+1})^{n-1}(u^{-1})^{n}\right], \qquad \text{(B.30)}
\end{aligned}
$$

where

$$
\mathcal{C}^{(n)\alpha\beta} \equiv (\overbrace{C_{11}\varepsilon C_{11}\cdots\varepsilon C_{11}}^{n})^{\alpha\beta} \qquad \text{(B.31)}
$$

and $\alpha^{(1,0)}_{n}, \beta^{(1,0)}_{n}$ are given in (B.11). To check this, we need (A.1) and (A.2).

Then from eq. (B.22) we find

$$
\lambda^{(0,1)}_{\dot{\alpha}} = -2\sqrt{2}i\bar{\xi}_{\dot{\alpha}i}\phi\;u^{-i} + \sum_{n=1}^{\infty}(-4i)(\bar{\xi}_{i}\bar{\sigma}^{\mu}\varepsilon\mathcal{C}^{(n)}\sigma^{\nu})_{\dot{\alpha}}A_{\mu}A_{\nu}(2\sqrt{2}\bar{\phi})^{n-1}\times \qquad \text{(B.32)}
$$

$$\times\left[\alpha_n^{(1,0)}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-1}\cdots u^{-1)}}^{n+1}+\beta_n^{(1,0)}\epsilon^{i1}(u^{+1})^{n-1}(u^{-1})^n\right]. \quad \text{(B.33)}$$

From eq. (B.25), we find $\delta^*_{\bar\xi}\bar\phi=\sqrt{2}i\bar\xi_i\bar\psi^i$ and

$$\lambda^{(2,0)}=-2\bar\xi_k\bar\psi_l u^{-(k}u^{-l)}\,. \quad \text{(B.34)}$$

In eq. (B.26), $\lambda_\mu^{(1,1)}$ is given by

$$\begin{aligned}
\lambda_\mu^{(1,1)} &= -2\psi_i\sigma_\mu\bar\xi_j\ u^{-(i}u^{-j)}+\\
&+\left(\lambda^{(1,1)}_{\mu\ ij\,(1)}+\tilde\lambda^{(1,1)}_{\mu\ ij\,(1)}\right)\left[2u^{+(1}u^{-i}u^{-j}u^{-1)}+\frac{3}{2}\Big\{\epsilon^{i1}u^{-(j}u^{-1)}+(i\leftrightarrow j)\ \Big\}\right]+\\
&+\sum_{n=2}^{\infty}\lambda^{(1,1)}_{\mu\ ij\,(n)}\Bigg[\alpha_n^{(1,1)}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-j}u^{-1}\cdots u^{-1)}}^{n+2}+\\
&\qquad+\beta_n^{(1,1)}\frac{1}{2}\Big\{\epsilon^{i1}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-j}u^{-1}\cdots u^{-1)}}^{n+1}+(i\leftrightarrow j)\ \Big\}+\\
&\qquad+\gamma_n^{(1,1)}\epsilon^{i1}\epsilon^{j1}(u^{+1})^{n-2}(u^{-1})^n+\delta_n^{(1,1)}\epsilon^{ji}(u^{+1})^{n-1}(u^{-1})^{n+1}\Bigg]+\\
&+\sum_{n=2}^{\infty}\tilde\lambda^{(1,1)}_{\mu\ ij\,(n)}\Bigg[\tilde\alpha_n^{(1,1)}\overbrace{u^{+(1}\cdots u^{+1}}^{n}\overbrace{u^{-i}u^{-j}u^{-1}\cdots u^{-1)}}^{n+2}+\\
&\qquad+\tilde\beta_n^{(1,1)}\frac{1}{2}\Big\{\epsilon^{i1}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-j}u^{-1}\cdots u^{-1)}}^{n+1}+(i\leftrightarrow j)\ \Big\}+\\
&\qquad+\tilde\gamma_n^{(1,1)}\epsilon^{i1}\epsilon^{j1}(u^{+1})^{n-2}(u^{-1})^n\Bigg], \qquad \text{(B.35)}
\end{aligned}$$

where

$$\begin{aligned}
\lambda^{(1,1)}_{\mu\ ij\,(n)} &\equiv -\frac{4}{3}(\bar\xi_{(i}\bar\sigma^\nu\varepsilon\mathcal{C}^{(n)}\sigma_\mu\bar\psi_{j)})A_\nu(2\sqrt{2}\bar\phi)^{n-1}\,,\\
\tilde\lambda^{(1,1)}_{\mu\ ij\,(n)} &\equiv \frac{2}{3}\mathcal{C}^{(n)\alpha\beta}(\sigma^\nu\bar\sigma_\mu\varepsilon)_{\alpha\beta}A_\nu(2\sqrt{2}\bar\phi)^{n-1}\bar\xi_{(i}\bar\psi_{j)}-\frac{1}{3}(\psi_{(i}\varepsilon\mathcal{C}^{(n)}\sigma_\mu\bar\xi_{j)})(2\sqrt{2}\bar\phi)^n\,, \qquad \text{(B.36)}
\end{aligned}$$

and

$$\begin{aligned}
(\alpha_2^{(1,1)},\beta_2^{(1,1)},\gamma_2^{(1,1)},\delta_2^{(1,1)}) &= \left(\frac{5}{4},\frac{10}{3},2,-\frac{1}{2}\right),\quad \alpha_n^{(1,1)}=\frac{1}{n+1}\left(6\alpha_{n-1}^{(1,0)}+\alpha_{n-1}^{(1,1)}\right),\\
\beta_n^{(1,1)} &= \frac{1}{n+1}\left(\frac{6n}{2n-1}\alpha_{n-1}^{(1,0)}+6\beta_{n-1}^{(1,0)}+\frac{1}{n}\alpha_{n-1}^{(1,1)}+\beta_{n-1}^{(1,1)}\right),\\
\gamma_n^{(1,1)} &= \frac{1}{n}\Bigg(\frac{6n}{(2n+1)(2n-1)}\alpha_{n-1}^{(1,0)}+3\beta_{n-1}^{(1,0)}+\\
&\qquad+\frac{n+1}{(2n+1)2n(2n-1)}\alpha_{n-1}^{(1,1)}+\frac{1}{2n-2}\beta_{n-1}^{(1,1)}+\gamma_{n-1}^{(1,1)}\Bigg),\\
\delta_n^{(1,1)} &= \frac{1}{n+1}\left(\frac{3(n-1)}{2n-1}\alpha_{n-1}^{(1,0)}-3\beta_{n-1}^{(1,0)}+\delta_{n-1}^{(1,1)}\right), \qquad \text{(B.37)}
\end{aligned}$$

$$
\begin{aligned}
(\tilde\alpha_2^{(1,1)},\tilde\beta_2^{(1,1)},\tilde\gamma_2^{(1,1)}) &= \left(\frac{1}{2},\frac{4}{3},\frac{4}{5}\right),\\
\tilde\alpha_n^{(1,1)} &= \frac{1}{n+2}\tilde\alpha_{n-1}^{(1,1)}, \qquad \tilde\beta_n^{(1,1)} = \frac{1}{n+1}\left(\frac{1}{n}\tilde\alpha_{n-1}^{(1,1)}+\tilde\beta_{n-1}^{(1,1)}\right),\\
\tilde\gamma_n^{(1,1)} &= \frac{1}{n}\left(\frac{n+1}{(2n+1)2n(2n-1)}\tilde\alpha_{n-1}^{(1,1)}+\frac{1}{2n-2}\tilde\beta_{n-1}^{(1,1)}+\tilde\gamma_{n-1}^{(1,1)}\right). \qquad \text{(B.38)}
\end{aligned}
$$

Substituting eq. (B.35) to the r.h.s. of eq. (B.26) and using (A.1), (A.2) and

$$
-\sqrt{2}C_{11}^{\alpha\beta}(\sigma_\mu\bar\sigma^\nu\varepsilon)_{\alpha\beta}\tilde\lambda^{(1,1)}_{\nu\ ij\,(n)}\bar\phi = \tilde\lambda^{(1,1)}_{\mu\ ij\,(n+1)}\,, \qquad \text{(B.39)}
$$

we can see that all the $O((C_{11})^2)$ terms are gauged away and the contributions to $\delta^*_\xi A_\mu$ come only from the $(C_{11})^1$ part of the second term besides the first term.

The $C^3$ term in (B.24) becomes

$$
+\frac{16\sqrt{2}}{3}\det C_{11}C_{11}^{\mu\nu}\partial_\mu\lambda^{(2,0)}\partial_\nu\bar\phi\;(u^{+1})^4(u^{-1})^2\,,
$$

which will be gauged away (because of the excessive number of $u^+$). As a result, we can regard eq.(B.24) as having the same structure as eq. (B.26) in terms of the harmonic variables. Therefore, it is solved in a way similar to (B.26): The $O((C_{11})^2)$ terms in the r.h.s. of eq. (B.24) are completely gauged away, and the relevant contributions to $\delta^*_\xi\phi$ are coming from each $O((C_{11})^0)$ part of $\lambda_\alpha^{(1,0)}$ and $\lambda_\mu^{(1,1)}$.

In eq. (B.28), $\lambda_{\dot\alpha}^{(2,1)}$ is given by

$$
\begin{aligned}
\lambda_{\dot\alpha}^{(2,1)} &= -2\bar\xi_{\dot\alpha(i}D_{jk)}u^{-(i}u^{-j}u^{-k)} - (\bar\xi_i\bar\sigma^\nu\varepsilon C_{11}\sigma^\mu)_{\dot\alpha}\partial_\mu(A_\nu\bar\phi)u^{-(i}u^{-1}u^{-1)} -\\
&\quad -\sum_{n=2}^{\infty}2(\bar\xi_i\bar\sigma^\nu\varepsilon\mathcal{C}^{(n)}\sigma^\mu)_{\dot\alpha}\partial_\mu\left\{A_\nu(2\sqrt{2}\bar\phi)^n\right\}\times\\
&\quad \times\left[\alpha_n^{(2,1)}\overbrace{u^{+(1}\cdots u^{+1}}^{n-1}\overbrace{u^{-i}u^{-1}\cdots u^{-1)}}^{n+2}+\beta_n^{(2,1)}\epsilon^{i1}(u^{+1})^{n-2}(u^{-1})^{n+1}\right], \qquad \text{(B.40)}
\end{aligned}
$$

where $\alpha_n^{(2,1)},\beta_n^{(2,1)}$ are given in (B.18). We can easily find that in the r.h.s. of (B.28) the terms proportional to $(C_{11})^n$ $(n\geq 2)$ are completely cancelled by this gauge parameter, so that there are no $O((C_{11})^2)$ terms in $\delta^*_\xi\bar\psi^i$.

Substituting (B.32), (B.30), (B.34), (B.35) and (B.40) into eq.(B.27), we can check that the gauge parameter $\lambda_\alpha^{(1,2)}$ having the same form as (B.19) is sufficient. To see it is sufficient, we need (A.1)–(A.4). In fact, in order to determine $\delta^*_\xi\psi^i$, only the precise forms of $\chi_{\alpha ijk(0)}$ and $\chi_{\alpha ij(1)}$ are needed as in the $\mathcal{N}=(1,0)$ case:

$$
\chi_{\alpha ijk(0)}=0\,,\quad \chi_{\alpha ij(1)}=\frac{32}{5}\left[D_{(ij}(\bar\xi_{k)}\bar\sigma^\mu\varepsilon C_{11})^\beta\varepsilon_{\alpha\beta}A_\mu+2i(\varepsilon C_{11}\psi_{(i})_\alpha\bar\xi_j\bar\psi_{k)}\right]\epsilon^{k1}. \qquad \text{(B.41)}
$$

Substituting (B.30), (B.34) and (B.35) to eq. (B.29) and collecting $u^+$-independent terms, we obtain $\delta^*_\xi D^{ij}$. We do not need the precise form of $\lambda^{(2,2)}$. To determine $\delta^*_\xi D^{ij}$, we can use (A.4) and (A.2).

The $\mathcal{N}=(0,1/2)$ supersymmetry transformation can be read from the following result by setting $\bar\xi^1=0$:

$$\delta^*_{\bar\xi}\phi = c_1 i(\bar\xi^1\bar\sigma^\mu\varepsilon C_{11}\psi^1)A_\mu\,, \tag{B.42}$$

$$\delta^*_{\bar\xi}\bar\phi = -\sqrt{2}i\bar\xi^i\bar\psi_i\,, \tag{B.43}$$

$$\begin{aligned}\delta^*_{\bar\xi}A_\mu &= i\bar\xi^i\bar\sigma_\mu\psi_i + c_2 i(\bar\xi^1\bar\sigma^\nu\varepsilon C_{11}\sigma_\mu\bar\psi^1)A_\nu + \\ &\quad + c_3 i C_{11}^{\alpha\beta}(\sigma^\nu\bar\sigma_\mu\varepsilon)_{\alpha\beta}\bar\xi^1\bar\psi^1 A_\nu + c_4 i(\bar\xi^1\bar\sigma_\mu\varepsilon C_{11}\psi^1)\bar\phi\,,\end{aligned} \tag{B.44}$$

$$\begin{aligned}\delta^*_{\bar\xi}\psi^{\alpha i} &= -\sqrt{2}(\bar\xi^i\bar\sigma^\mu)^\alpha\partial_\mu\phi - 2D^{(i1}(\bar\xi^{1)}\bar\sigma^\mu\varepsilon C_{11})^\alpha A_\mu + 4i\bar\xi^{(i}\bar\psi^1(\psi^{1)}\varepsilon C_{11})^\alpha + \\ &\quad +\delta^i_2\Big[c_5(\bar\xi^1\bar\sigma^\mu\varepsilon C_{11})^\alpha\partial_\mu\phi\bar\phi + (\bar\xi^1\bar\sigma^\mu\varepsilon C_{11}\sigma^\nu\bar\sigma^\rho)^\alpha\Big\{c_6\partial_\rho(A_\mu A_\nu) + c_7 A_\mu\partial_\nu A_\rho\Big\} + \\ &\qquad + c_8(\bar\xi^1\bar\sigma^\mu)^\alpha C_{11}^{\beta\gamma}(\sigma^\nu\bar\sigma^\rho\varepsilon)_{\beta\gamma}\partial_\rho A_\mu A_\nu\Big] + \\ &\quad + \delta^i_2\det C_{11}\left[(\bar\xi^1\bar\sigma^\mu)^\alpha\left\{c_9 D^{11}A_\mu\bar\phi + c_{10}i\bar\psi^1\bar\psi^1 A_\mu\right\} + c_{11}i\bar\xi^1\bar\psi^1\psi^{\alpha 1}\bar\phi\right],\end{aligned} \tag{B.45}$$

$$\delta^*_{\bar\xi}\bar\psi_{\dot\alpha i} = (\bar\xi_i\bar\sigma^{\mu\nu})_{\dot\alpha}F_{\mu\nu} - \bar\xi^j_{\dot\alpha}D_{ij} + \delta^1_i c_{12}\partial_\mu\left[(\bar\xi^1\bar\sigma^\nu\varepsilon C_{11}\sigma^\mu)_{\dot\alpha}A_\nu\bar\phi\right], \tag{B.46}$$

$$\begin{aligned}\delta^*_{\bar\xi}D^{11} &= c_{13}i\partial_\mu(\bar\xi^1\bar\sigma^\mu\psi^1),\\ \delta^*_{\bar\xi}D^{12} &= -2i\partial_\mu(\bar\xi^{(1}\bar\sigma^\mu\psi^{2)}) + \\ &\quad + i\partial_\mu\Big[c_{14}(\bar\xi^1\bar\sigma^\nu\varepsilon C_{11}\sigma^\mu\bar\psi^1)A_\nu + c_{15}C_{11}^{\alpha\beta}(\sigma^\nu\bar\sigma^\mu\varepsilon)_{\alpha\beta}A_\nu\bar\xi^1\bar\psi^1 + c_{16}(\bar\xi^1\bar\sigma^\mu\varepsilon C_{11}\psi^1)\bar\phi\Big],\\ \delta^*_{\bar\xi}D^{22} &= -2i\partial_\mu(\bar\xi^2\bar\sigma^\mu\psi^2) - \frac{4}{3}i\partial_\mu \times \\ &\quad \times\Big[2(\bar\xi^{(1}\bar\sigma^\nu\varepsilon C_{11}\sigma^\mu\bar\psi^{2)})A_\nu - C_{11}^{\alpha\beta}(\sigma^\nu\bar\sigma^\mu\varepsilon)_{\alpha\beta}A_\nu\bar\xi^{(1}\bar\psi^{2)} + \sqrt{2}(\bar\xi^{(1}\bar\sigma^\mu\varepsilon C_{11}\psi^{2)})\bar\phi\Big] + \\ &\quad + i\det C_{11}\partial_\mu\left[c_{17}(\bar\xi^1\bar\sigma^\nu\sigma^\mu\bar\psi^1)A_\nu\bar\phi + c_{18}\bar\xi^1\bar\psi^1 A^\mu\bar\phi + c_{19}(\bar\xi^1\bar\sigma^\mu\psi^1)\bar\phi^2\right],\end{aligned} \tag{B.47}$$

where $c_i$'s are certain constants.

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